

# Time and the Price Impact of a Trade

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## ABSTRACT

We use Hasbrouck's (1991) vector autoregressive model for prices and trades to empirically test and assess the role played by the waiting time between consecutive transactions in the process of price formation. We find that as the time duration between transactions decreases, the price impact of trades, the speed of price adjustment to trade-related information, and the positive autocorrelation of signed trades all increase. This suggests that times when markets are most active are times when there is an increased presence of informed traders; we interpret such markets as having reduced liquidity.

THE AVAILABILITY OF LARGE data sets on transaction data and powerful computational devices has generated a new wave of interest in market microstructure research and has opened new frontiers for the empirical investigation of its hypotheses. The microstructure literature is mainly devoted to the study of the mechanics of price formation, examining questions such as, "What are the determinants of the behavior of prices?" and "How is new information incorporated into prices?"<sup>1</sup> Hasbrouck's (1991) analysis reveals that the change in prices depends on characteristics of trades (sign and size) and the market environment as measured by bid-ask spread, in addition to the current and past levels of prices. At time  $t$ , when a trade is performed, larger transaction size and spread imply a larger price revision after the trade. Furthermore, not only does volume affect prices, but it has a persistent impact on prices, which means volume conveys information. The intuition, supported by theoretical predictions, is that other trade-related variables might also be informative. The primary objective of this paper is to provide empir-

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<sup>1</sup> We refer the interested reader to O'Hara (1995) and Hasbrouck (1996). These are two major efforts in trying to synthesize the existing and fast growing literature on market microstructure.

ical evidence that the time between trades, which is a measure of trading activity, affects market price behavior. Moreover, by measuring how much and how fast prices respond to trades at any point during the trading day, we illuminate the dynamic behavior of some aspects of market liquidity that could be used to design optimal trading strategies.

The background of this study goes back to Bagehot (1971), who first considered a scenario with heterogeneously informed traders. According to Bagehot, the specialist, possessing only publicly available information, faces informed and uninformed traders, but cannot distinguish between them. Uninformed traders are also called liquidity traders because their trading is either motivated by consumption needs and portfolio strategies or simply reflects personal price sensitivity, or specific trading rules (Easley and O'Hara (1992)). The market maker fixes a spread that compensates on average for the losses suffered from trading with the informed. These are the basic elements of the asymmetric information models that were introduced by Bagehot (1971), analyzed by Copeland and Galai (1983), and formalized and developed by Kyle (1985), Glosten and Milgrom (1985), Easley and O'Hara (1987), Admati and Pfleiderer (1988), Foster and Viswanathan (1990, 1994), and Allen and Gorton (1992).

A key element of asymmetric information models is that trades convey information. The specialist, by observing trading activity, *gradually* learns the information held by informed traders and adjusts prices so that, at any point in time, they reflect the expectation of the security terminal value conditional on all public information, including prior trades. As a consequence, only in the long run will prices fully incorporate the new information. Price dynamics are therefore derived from the mechanism of market-maker learning. For this purpose, Hasbrouck (1991, 1996) focuses on the study of the price effect of a trade and identifies a short-run effect and a long-run effect. Hasbrouck (1991) provides empirical evidence showing that the market maker, through the observation of trade attributes such as sign and size, infers information from the sequence of trades. We extend Hasbrouck's results in two main directions. First, we test the informational role of market activity measured by the time interval between two consecutive transactions, and second, we show how to exploit the information content of time durations to enhance models for price and trade dynamics.

The theoretical motivations for our empirical investigation on the role of time between trades are found in the models of Diamond and Verrecchia (1987) and Easley and O'Hara (1992). Prior to these two contributions, the market microstructure literature did not accord time a prominent informational role.<sup>2</sup> In Diamond and Verrecchia (1987), at the beginning of the trading day, one of two possible events happens, either good news or bad news.

<sup>2</sup> In Kyle (1985), orders are accumulated and executed together at a specific point in time and at a single price so that the order arrival is not important. In Glosten and Milgrom (1985), orders arrive according to some exogenous probabilistic process known to the specialist. Only Garman (1976) and Garbade and Lieber (1977) indirectly hint at time's significance.

Thus, informed traders will always trade unless they do not own the stock and short-sale constraints exist. Accordingly, long durations are likely to be associated with bad news. In Easley and O'Hara (1992), informed traders trade on either side of their signal, but only when there *is* a signal ("news") and therefore long durations are likely to be associated with no news. These two contributions suggest that time actually conveys information. By definition, an uninformed trader's decision to trade is independent of the existence of any information. However, informed traders only trade when they have information, hence variations in trading rates in Easley and O'Hara (1992) are associated with changing numbers of informed traders. More generally, informed traders would presumably choose to trade as quickly as possible and as much as possible once they have received their information. However, as analyzed by Easley and O'Hara (1987), informed traders may be quickly distinguished by their large volume trading and hence their profit opportunities would be lessened. Thus the incentives to trade quickly are reduced. On the other hand, informed traders may choose to break up large volume trades, thereby generating a larger number of informationally based trades. Thus, it is reasonable to assume that variations in the trading intensity are positively related to the behavior of informed traders. Therefore trading intensity, which results in short and long durations between trades, may provide information to market participants.

The theoretical models above formulate a plausible role for time, but we agree with O'Hara that "the importance of time is ultimately an empirical question . . ." (O'Hara (1995)). Even though ours is not the first attempt to empirically test the importance of time, no prior work, to our knowledge, has definitely assessed the role that time plays in the process of price dynamics. Hasbrouck (1991) suggests, as a promising idea for a better understanding of the price impact of a trade, the study of time-of-day patterns in trade impact on prices. Hausman, Lo, and MacKinlay (1992) estimate ordered probit models for transaction prices with time between the current and the last trade as an explanatory variable. From this analysis, time seems to matter, but the interpretation of the role of time is inconclusive. The importance of time is confirmed by two other studies on transaction data for the IBM stock: (1) Engle and Russell (1994) find evidence of comovements among duration, volatility, volume, and spread, and (2) Engle (1996) observes that longer (shorter) durations lead to lower (higher) volatility. Thus trading intensity is correlated with volatility. Our analysis is similar in objective to Easley, Kiefer, and O'Hara (1993). However, their method for extracting information from the daily number of no-trade intervals relies on assumptions, such as the independence of trades, that are difficult to reconcile with empirical results on intraday data.

The original contribution of this paper lies in providing strong empirical evidence of the relevance of time in the process of price adjustment to information, which confirms Easley and O'Hara's predictions. Our investigation reveals that the time between trades is informative, and higher trading intensity is associated with a higher price impact of trades, a faster price

adjustment to new trade-related information, and a stronger autocorrelation of trades. To describe a general pattern, we base our results on a large sample composed of 18 of the most frequently traded stocks on the New York Stock Exchange (NYSE) covered by the TORQ data set. Furthermore, we relate this result to the existing microstructure literature on time, and describe the important role of time in a model of price formation. In this framework, we use the data to shed light on the theoretically ambiguous relation between transaction rate and liquidity traders' behavior. On one hand, high trading intensity is caused by informed and uninformed traders pooling together to trade at low costs (see Admati and Pfleiderer (1988)). Alternatively, high activity is due to informed traders whose presence on the market deters the uninformed from trading (see Foster and Viswanathan (1990)). Only this latter view is, in fact, consistent with our findings.

Our results have relevant practical implications for studying the liquidity process. In general, liquidity is related to the ease with which securities are bought and sold without wide price fluctuations. The literature, however, recognizes the complexity of liquidity and identifies four dimensions: width, depth, immediacy, and resiliency (Harris (1990)). Grossman and Miller (1988) and Brennan and Subrahmanyam (1996) criticize earlier studies that focus on spread as a liquidity-cost measure, neglecting the dynamic aspect of liquidity. In Seppi (1997), market liquidity is related to the *temporary* or *non-informational* price impact of different sizes of market orders. In the context of asymmetric information models, liquidity suppliers (specialist and limit orders) do not know whether the trade initiator is motivated by private information or exogenous needs. Therefore, in our empirical analysis, we define a liquid market as a market in which trades have a lower impact on prices and, consequently, new trade-related information takes longer to be fully incorporated into prices. Thus, using a general model for price dynamics to measure the value and the speed of price change after trades is a natural framework for studying the liquidity process and assessing the level of liquidity risk.<sup>3</sup>

The paper is organized as follows. Section I presents Hasbrouck's (1991) framework and explains the generalization used in our study. Section II describes the data and variables used in the analysis. Section III develops the estimation techniques employed and discusses the results of our study, while focusing on the relation between time and the price impact of trades. Finally, Section IV summarizes and concludes.

## I. The Model

Prices,  $q_t$ , are measured by the average of the bid and the ask quotes just prior to the  $t$ th trade,  $x_t$ . We adopt the common convention of using  $t$  to index trades. Prices are often modeled as the sum of an informationally

<sup>3</sup> Biais, Hillion, and Spatt (1995) investigate the relation between trading intensity and the supply of liquidity in the Paris Bourse.

efficient price disturbed by transitory perturbations. The usual assumption is that the informational component of prices follows a nonstationary process with a unit root and, consequently, we study price variations,  $\Delta q_t$ . Note that, in agreement with previous literature (see Hasbrouck (1991)),  $\Delta q_t$  is conventionally defined as the quote change subsequent to the  $t$ th trade. Furthermore, the informational component of price variation can be related to two different sources of information, public and private. These informational shocks are commonly represented with two white noise processes  $v_{1,t}$  and  $v_{2,t}$ . Specifically,  $v_{1,t}$  is the update to the public information set and  $v_{2,t}$  is the update from the private information, which is gleaned from unexpected trades.

### A. Hasbrouck Model

Hasbrouck (1991) suggests the following *vector autoregression* (VAR),

$$\begin{aligned}\Delta q_t &= \sum_{i=1}^{\infty} a_i \Delta q_{t-i} + \sum_{i=1}^{\infty} b_i x_{t-i} + v_{1,t} \\ x_t &= \sum_{i=1}^{\infty} c_i \Delta q_{t-i} + \sum_{i=1}^{\infty} d_i x_{t-i} + v_{2,t},\end{aligned}\tag{1}$$

to study the effects of trade-related information on prices.<sup>4</sup> This general model for changes in quotes and trade dynamics includes as special cases many of the microstructure models introduced in the literature. We consider the simplest version of this model where  $x_t$  is a univariate limited dependent variable, the trade sign. Hasbrouck (1991) also proposes generalizations with  $x_t$  as a vector of trade-related variables (e.g., trade sign, the interaction between trade sign and volume, the interaction between trade sign and spread). We assume that summations in model (1) can be truncated at five lags and the model can be estimated consistently by ordinary least squares (OLS).<sup>5</sup> Conjecturing that public information previous to the  $t$ th trade does not help forecast future trade innovations,<sup>6</sup> an estimable version of model (1) is obtained with the current trade in the first equation and further assuming that  $v_{1,t}$  and  $v_{2,t}$  are jointly and serially uncorrelated with zero mean.

While it is unusual to have a limited dependent variable in a vector autoregression, this presents no econometric difficulties when it is an explanatory variable, which is the case for the relation of primary interest, the price equation. When estimating the trade equation, the linear specification is potentially inappropriate. Nevertheless, OLS estimation still yields consis-

<sup>4</sup> de Jong, Nijman, and Röell (1995) have employed this model to study the price effect of trading on the Paris Bourse.

<sup>5</sup> Covariance stationarity of the price-trade process is typically assumed (see Hasbrouck (1991) and de Jong et al. (1995)).

<sup>6</sup> See Hasbrouck (1991) for the justification of this identifying hypothesis.

tent parameter estimates, if the conditional mean of the trade indicator is correctly specified. Because the probability of a buy or sell never is far from 1/2, the linear specification is likely to be adequate. Nonetheless, analogously to the case of the *linear probability* model for binary dependent variables, the estimates are inefficient and standard errors are biased. We correct the standard errors by using White's (1980) heteroskedasticity consistent covariance estimator to construct Wald and *t*-statistics.

Our main concern is a better understanding of the price impact of a trade. The immediate price effect of the current trade is measured by the  $b_0$  coefficient. However, as discussed by Hasbrouck (1991, 1996), there are permanent and transitory effects on prices. Both of these effects can be potentially influenced by all of the coefficients in model (1). Hasbrouck (1991) proposes to compute the permanent price effect of a trade by calculating the impulse response function from the *vector moving average* (VMA) representation:

$$\begin{bmatrix} \Delta q_t \\ x_t \end{bmatrix} = \begin{bmatrix} \alpha(L) & \beta(L) \\ \gamma(L) & \delta(L) \end{bmatrix} \begin{bmatrix} v_{1,t} \\ v_{2,t} \end{bmatrix}, \quad (2)$$

where  $L$  is the lag operator. The impact of an unexpected trade on prices after  $k$  transactions,  $\Delta q_{t+k}^x$ , will be measured by the sum of the coefficients of the impulse response function

$$\Delta q_{t+k}^x = \sum_{i=0}^k \beta_i v_{2,t}. \quad (3)$$

Hence, the long-run impact of a trade on prices,  $\Delta q_{t+\infty}^x$ , is given by the limit of the summation in equation (3) for  $k \rightarrow \infty$ .

The approach presented above is very attractive for its relative simplicity and generality. It allows Hasbrouck to successfully model the dynamic interrelation of trade sign, trade volume, spread, and quote revision, with the following main results. When the transaction size is larger and the spread is wider, the quote revision after the trade is also larger. Furthermore, quote revision and volume have a nonlinear concave relation. We propose to expand this model by including the time between consecutive transactions as an additional determinant of the price impact of a trade.

### B. Generalizing with Time

In terms of understanding the role of time in security markets, we are still at the beginning. From an economic point of view, we have the following situation. The driving forces of model (1) are private and public information processes that evolve in time in a nonhomogeneous fashion.<sup>7</sup> Quote setters,

<sup>7</sup> See Berry and Howe (1994) for a study of the effects of public information arrival on market activity.

broadly defined to include market makers and limit order traders, post schedules conditional on their information, which includes the history of trades and quotes and more generally is identified with the public information set. Quote setters are assumed to react immediately to the release of new public information. The signed trade generation process, instead, is driven by the information processes, liquidity needs, and, in some models, a random selection mechanism that determines who has the opportunity to trade.

Theoretical analyses such as Garman (1976), Easley and O'Hara (1992), and Diamond and Verrecchia (1987) claim a role for the process of trade arrival times in models of market microstructure. Explicitly modeling this time process, though, is not an easy task. It requires modeling not only traders' behavior and, in particular, how much of the information known by the insiders has been made public (Glosten and Milgrom (1985)), but also any friction and imperfection that may be present in the trading mechanism (e.g., program trading) (Easley and O'Hara (1992)). It is common to take these arrival rates as exogenous, at least as a first approximation.

The assumption that the arrival process of trades is exogenous, or more precisely, according to Engle, Hendry, and Richard (1983), strongly exogenous, implies that optimal forecasts of the arrival of trades depend only on past arrival times and the time of the day. Prices and quotes are assumed not to influence the arrival of trades. One can certainly visualize situations where price movements would influence the arrival rates of trades. For example, a sudden drop in the asking price could call forth a series of market buy orders and thus increase the transaction rate. Similarly, a wider spread should reduce the transaction rate all else being equal. The empirical importance of these effects is not clear.

In this paper, we maintain the exogeneity assumption for the time process, treating intertrade time durations  $\{T_t\}$  as strongly exogenous to both the price and trade processes. We also present evidence questioning the validity of this assumption in Section III.E. The probabilistic structure for trade arrivals is modeled with a point process for irregularly spaced data, such as Engle and Russell's (1998) Autoregressive Conditional Duration model.

We proceed by extending model (1) to allow the trade coefficients to vary with time, where time is considered to have a deterministic component (time of day) plus random variations. Our objective is to investigate if the time between trades  $T_t$  (where  $T$  is measured in seconds) affects the price adjustment to trades in the return equation, and affects the correlation between current and past trades in the signed trade equation. We adopt the simple bivariate model for trades and quote revisions in model (1), introducing time as a predetermined variable that influences both the price impact of a trade and the correlation between trades. Defining  $r_t$  as the change in the logarithm of midquote prices, the quote revision equation can be written as

$$r_t = \sum_{i=1}^5 a_i r_{t-i} + \sum_{i=0}^5 [\gamma_i^r + \mathbf{z}_{t-i} \boldsymbol{\delta}_i^r] x_{t-i}^0 + v_{1,t}, \quad (4)$$

where  $x_t^0$  is the trade sign and  $\mathbf{z}_{t-i}$ , and  $\delta_i$  are vectors. The quantity in square brackets, which replaces  $b_i$  in model (1), is the parameterization for the trade impact on quote revision. For  $i = 0$ , it measures the immediate effect of a signed trade on the specialist's quote. In particular,  $\mathbf{z}_{t-i}$  is a row vector of observations for the determinants of the trade impact, and  $\delta_i$  is the column vector of corresponding coefficients. Because we are interested in studying the role of time and in particular if it has any distinguishable effects from the time-of-day effects,<sup>8</sup> we initially consider the case where the only variables in  $\mathbf{z}_t$  are current and past time durations,  $T_{t-i}$ , and a set of time-of-day dummy variables,  $D_{j,t-i}$ . We then specify the trade impact as

$$b_i = \gamma_i^r + \sum_{j=1}^J \lambda_{j,i}^r D_{j,t-i} + \delta_i^r \ln(T_{t-i}), \quad (5)$$

where intertransaction time enters nonlinearly in the dynamic model for price changes and trades.<sup>9</sup> If all the  $\delta$ s and  $\lambda$ s in equation (5) are jointly zero, then equation (4) is the same as the first equation in model (1). Moreover, if  $\delta_i^r$ ,  $i = 0, \dots, 5$  in equation (5) are jointly zero and at the same time at least one  $\lambda_{j,i}^r$ ,  $i = 1, \dots, 5$ ;  $j = 1, \dots, J$  is not zero, then the trade impact on prices exhibits only intraday periodicities.

The trade equation can be similarly modified to examine own-variable effects of trade durations on trades. In particular, if  $x_t^0$  replaces  $x_t$  in model (1), we can think of the  $d_i$  coefficients as also being time varying and having, for example, a parameterization analogous to equation (5)

$$d_i = \gamma_i^x + \sum_{j=1}^J \lambda_{j,i}^x D_{j,t-i} + \delta_i^x \ln(T_{t-i}). \quad (6)$$

In system (1), the impact of a 10,000-share transaction is the same regardless of what time during the trading day it is performed. In contrast, in equation (5) the impact of a trade on the subsequent quote revision varies

<sup>8</sup> Intraday periodicities have been found to characterize the behavior of several market variables: public information arrival (Berry and Howe (1994)), trading volume (Jain and Gun-Ho (1988) and Foster and Viswanathan (1993)), return (Harris (1986) and Wood, McInish, and Ord (1985)), and time durations (Engle and Russell (1998)).

<sup>9</sup> Since  $T_t$  can indeed be zero, without loss of generality we add one second to each duration. A further consideration is worthwhile. Hasbrouck (1991) finds that the trade impact on prices is positive and persistent. Easley and O'Hara (1992) predict that this impact will be lower for trades arriving after a long period of inactivity. Therefore,  $\gamma_i$ s are presumably positive,  $\delta_i$ s are negative and such that the net effect is positive. However, in our parameterization, the negative duration effect may still increase without bounds and therefore trades arriving after a sufficiently long duration may potentially have a negative price effect. To avoid this, we would need a nonlinear specification for the time effect that also allows for some saturation, for example  $b_i = \gamma_i + \dots + \delta_i \exp(-\kappa T_{t-i})$ . Our experience with such parameterization is that it offers greater flexibility, but at the same time it requires more computation in the estimation process, and often the interpretation of the parameters is not easy. For example, when testing for the significance of time durations in the trade impact specification, the distribution of  $\kappa$  is not known under the null hypothesis of  $\delta_i$  jointly zero.

with the time of the day and the level of trading intensity, measured by the time between consecutive trades. Analogously, in equation (6) we allow the correlation between trades to vary throughout the trading day. With regard to the daily component of both the trade impact on prices and the autocorrelation of trades, we conduct thorough experimentation on the explanatory power of lagged diurnal dummies as we describe at the beginning of Section III. The results strongly indicate that equations (5) and (6) can be greatly simplified by excluding lagged diurnal dummies.

The relevance of time in the dynamic relationship of quote revisions and trades can be verified by testing if the coefficients,  $\delta_i$ , are significantly different from zero. Moreover, according to previous empirical findings, a general specification for the price impact of a trade will also include in  $z_{t-i}$  trade size and spread. We therefore test the specific contribution of time when these other determinants are included in  $z_{t-i}$ .

Before turning to the data and the empirical analysis, we have to complete the description of our model with the introduction of a process for time durations. As we discussed above, price and trade dynamics are modeled conditional on the process of time durations and therefore the modified VAR specification can be directly estimated. However, when the joint density of prices, trades, and time durations is needed, for example to compute the impulse response function for the generalized system, we have to explicitly specify the Data Generation Process for time durations.

### C. A Model for Intertrade Arrival Times

In the econometric literature, two different approaches have been taken to modeling irregularly spaced data: *Time Deformation* (TD) models (see, for example, Clark (1973), Stock (1988), and Ghysels and Jasiak (1994)) and *Autoregressive Conditional Duration* (ACD) models (see Engle (1996) and Engle and Russell (1998)). The TD approach uses auxiliary transformations (see Stock (1988)) to relate observational/economic time to calendar time. Instead, we prefer the ACD approach that directly models the time between events (e.g., trades). The ACD is a type of *dependent point process*<sup>10</sup> particularly suited for modeling characteristics of duration series such as clustering and overdispersion. We initially remove the deterministic diurnal component  $\Phi_{t-1}$  of intertrade arrival times and we consider the diurnally adjusted series of time durations  $\tilde{T}_t = T_t/\Phi_{t-1}$ . The ACD model consists first, of a distributional assumption for the conditional density of adjusted durations  $g(\tilde{T}_t)$ . The Weibull distribution is a plausible assumption and we can thereby write

$$g(\tilde{T}_t) = \frac{\theta}{\phi_t^\theta} \tilde{T}_t^{\theta-1} \exp \left[ - \left( \frac{\tilde{T}_t}{\phi_t} \right)^\theta \right] \quad \text{for } \theta, \phi_t > 0. \quad (7)$$

<sup>10</sup> For a recent survey, stressing the duality of Poisson processes for counts and model for durations, see Cameron and Trivedi (1996).

The exponential is a convenient and particular alternative. It is obtained as a special case of equation (7) when the scale parameter  $\theta = 1$ . The Weibull distribution is to be preferred if the data show overdispersion with extreme values (very short and long durations) more likely than the exponential would predict. In our sample this is clearly indicated by estimated  $\theta$ s lower than unity. Second, we need a specification for the  $t$ th duration conditional mean  $E(\tilde{T}_t | \tilde{T}_{t-1}, \dots, \tilde{T}_1) \equiv \phi_t \Gamma(1 + 1/\theta)$ , where  $\Gamma(\cdot)$  is the gamma function. For example,  $\theta_t$  may have the following linear ARMA-type parameterization

$$\phi_t = \omega + \sum_{j=1}^p \rho_j \tilde{T}_{t-j} + \sum_{i=1}^q \zeta_i \phi_{t-i} + \lambda D_{t-1}. \quad (8)$$

$D_t$  represents exogenous and dummy variables that may affect the conditional mean duration. For example, we include a dummy variable for the first observation of the trading day to correct for interday variations. Finally, we assume that standardized durations  $\epsilon_t \equiv (\tilde{T}_t/\phi_t)^\theta$ , which have unit exponential distribution, are i.i.d. for every  $t$ . This testable assumption ensures that all the temporal dependence in the duration series is captured by the mean function (Engle (1996)). The ACD models have a close resemblance to GARCH models and share many of their properties. Having specified a conditional density, it is straightforward to estimate the parameters  $\{\omega, \rho_j, \zeta_i, \theta, \lambda; j = 1, \dots, p, i = 1, \dots, q\}$  maximizing the following log-likelihood function

$$L(\boldsymbol{\eta}) = \sum_{t=1}^N l_t = \sum_{t=1}^N \ln \left( \frac{\theta}{\tilde{T}_t} \right) + \theta \ln \left( \frac{\tilde{T}_t}{\phi_t} \right) - \left( \frac{\tilde{T}_t}{\phi_t} \right)^\theta,$$

where  $\phi_t$  is specified as in equation(8) and  $\boldsymbol{\eta}$  is a column vector containing the parameters of interest. Outer-product-of-the-gradient (OPG) estimators<sup>11</sup> are found that are consistent for the parameters

$$(\hat{\boldsymbol{\eta}} - \boldsymbol{\eta}_0) \overset{A}{\sim} N(0, (\mathbf{G}^T(\hat{\boldsymbol{\eta}})\mathbf{G}(\hat{\boldsymbol{\eta}}))^{-1}),$$

where  $\mathbf{G}(\hat{\boldsymbol{\eta}})$  is a matrix with typical element  $G_{ti} \equiv \partial l_t / \partial \eta_i |_{\eta = \hat{\boldsymbol{\eta}}}$ .

## II. The Data

We use transaction data extracted from the TORQ (Transaction Orders and Quotes) database compiled by Hasbrouck (1992). The TORQ database contains information relative to transactions, orders, and quotes for 144 stocks

<sup>11</sup> The OPG estimator, also called the BHHH estimator from the initials of its original advocates (see Berndt et al. (1974)), is a byproduct of a computationally efficient algorithm for the maximization of the likelihood function. It is used in this paper, despite the well-known poor small sample performances (Davidson and MacKinnon (1993), p. 447), because such a problem is alleviated by the very large size of our data series.

**Table I**  
**Summary Statistics**

The data includes trades and quotes for 18 of the most actively traded stocks in the NYSE's TORQ database. Averages are computed over 62 trading days from November 1, 1990 to January 31, 1991. Quotes are the prevailing quotes at the time the trade is executed. Price is midquote price. Spread is given by the difference between ask and bid price. Duration is measured in seconds. Size is measured in number of shares. Ticker symbols and company names are provided in the first two columns.

Stock	Company Name	N	Average Price	Average Size	Average Spread	Average Duration
BA	Boeing Co.	38305	45.64	1617.39	0.180	37.08
CAL	Calfed Inc.	5674	3.30	1844.96	0.158	230.62
CL	Colgate-Palmolive Co.	8068	70.45	1286.02	0.226	169.56
CPC	CPC International Inc.	6388	78.31	1399.34	0.237	211.15
DI	Dresser Industries Inc.	9517	20.43	2119.26	0.198	144.39
FDX	Federal Express Corp.	5947	34.27	2086.50	0.234	225.99
FNM	Federal National Mortgage Association	28876	33.86	2944.03	0.163	48.93
FPL	FPL Group Inc.	19500	28.42	1037.47	0.153	72.69
GE	General Electric Co.	71080	56.00	1246.46	0.162	20.08
GLX	Glaxo Holdings PLC ADR	21223	32.79	1982.47	0.166	66.19
HAN	Hanson PLC ADR	7439	18.41	3231.19	0.152	186.56
IBM	International Business Machines Corp.	57882	114.22	1679.81	0.176	24.65
MO	Philip Morris Companies Inc.	59980	50.54	1753.57	0.152	23.68
NI	NipSCO Industries Inc. Holding Co.	6052	18.68	1245.85	0.173	219.01
POM	Potomac Electric Power Co.	9506	20.15	1104.97	0.153	145.50
SLB	Schlumberger LTD	14984	55.20	2016.42	0.245	93.44
T	American Telephone and Telegraph Co.	66832	30.95	1419.26	0.158	21.40
XON	Exxon Corp.	27246	50.62	2033.95	0.160	52.23

recorded on the tape of the NYSE computer system in the three months from November 1, 1990 to January 31, 1991. The 62 day trading sample is long enough to allow reasonably precise estimations (Easley, Kiefer, and O'Hara (1993) and Easley et al. (1996)). In our analysis, we consider price of trades, time stamp of trades, size of trades, and quotes for 18 of the most transacted stocks on the first day of the sample.<sup>12</sup> Summary statistics for the chosen stocks are presented in Table I. Notice the very low average price of Calfed (CAL) compared to the average price of the other stocks. If we look at the average duration, Calfed also appears to be the least frequently traded stock, and General Electric (GE), with an average transaction rate of one transaction every 20 seconds, is the most frequently traded.

The details of the data adjustments, the description of the new variables, and some preliminary insights from correlation analysis are presented in the following subsections.

<sup>12</sup> The names of the selected stocks appear in Table III. Note that among these stocks there are two American Depository Receipts (ADR); these are receipts representing shares of foreign corporations held by U.S. depository institutions which are quoted in U.S. dollars and trade just like any other stock.

### A. Preparation of the Data for the Analysis

We prepare the data for our analysis as follows. First, we filter out anomalous data. All transactions and quotes that occurred on November 23, 1990 are discarded because they reflect a very peculiar trading day pattern (the NYSE internal information computerized system had been down for a few hours that day and it is the day after Thanksgiving). In addition, we identify and eliminate reporting errors for the Boeing (BA) and GE series of bid and ask quotes.<sup>13</sup> Second, we keep only New York's quotes because they are the most representative of the prevailing consolidated prices (see Hasbrouck (1991, 1995) and Blume and Goldstein (1997) for empirical evidence). Third, we add up the size of consecutively recorded trades that are performed on the same regional exchange, with the same time stamp and the same price, and we treat them as one trade. Fourth, the data are viewed in transaction time, and for every transaction, the prevailing quote is the last quote that appears at least five seconds before the transaction itself. This is a conventional procedure proposed by Lee and Ready (1991) to correct reporting errors in the sequence of trades and quotes. Fifth, we omit the opening trade and the overnight price change to avoid contamination of prices by overnight news arrival. Thus we drop transactions occurring before the first quote and treat the overnight price change as a missing value for all lagged variables. Sixth, we also drop the transactions occurring after 4:00 p.m., the official closing time.<sup>14</sup> Despite these adjustments the sample remains very large, as it appears from Table I; for example, there are more than 70,000 observations in each data series for GE.

### B. Variables

We define  $r_t$  as the change in the natural logarithm of the midquote price that follows the current trade at time  $t$ ,  $r_t = 100(\ln(q_{t+1}) - \ln(q_t))$ . It is interesting to notice that the great majority of the time, the specialist does not revise the quotes after a trade. This results in an  $r_t$  series with a high percentage of zeros from a minimum of 72 percent for Colgate-Palmolive (CL) to a maximum of 96 percent for ATT (T). Time duration,  $T_t$ , is the difference in seconds between the time stamp for the trade  $x_t$  and the previous trade  $x_{t-1}$ . Without loss of generality we add one second to the whole series of durations, so that the lowest log-duration is still zero.

We use a dummy variable,  $x_t^0$ , which Hasbrouck (1991) calls the *trade indicator*. This variable assumes a value of one if a trade is initiated by a buyer, a value of minus one if a trade is initiated by a seller, and zero if the

<sup>13</sup> Summary statistics for the variables to be used in the estimation process reveal the presence of a limited amount of errors. These errors can be easily identified and eliminated without requiring any general filtering criteria.

<sup>14</sup> We also noted that the market closed early at 2:00 p.m. on December 24 and opened late at about 11:00 a.m. on December 27 when the first recorded trade is for GE at 10:42 a.m. We therefore adjusted the diurnal dummy variables accordingly.

trade is just a match of two opposite orders at the midquote. To create an approximate trade variable, as we do not have any information on the actual identity of the trade initiator, we adopt a classification rule known as the midquote rule (see Lee and Ready (1991)). We classify a trade as a purchase if the transaction price is higher than the midquote price, as a sale (trade initiated by a seller) if the transaction price is lower than the midquote price, or as unidentified if the transaction price is equal to the midquote price.<sup>15</sup> This procedure results in classifying, on average per stock, 35.82 percent of transactions as sales, 44.81 percent as buys, and leaves a residual average of 21.50 percent of unclassified trades. The predominance of buys over sells is remarkable for FPL Group (FPL), Nipsco Industries (NI), and Potomac Electric Power (POM), where more than 60 percent of the trades are classified as buys.

Before presenting the estimation results, we examine simple correlations among the relevant variables: absolute value of returns, spread, lagged time durations, and lagged trade size. Logarithmic time durations,  $\log(T_t)$ , are negatively correlated with the absolute value of the subsequent quote revision for 16 of the 18 stocks considered. The spread, defined as the difference between ask and bid prices divided by their average, is also negatively correlated with lag duration. This is compatible with a market maker that reduces the spread after a long interval of no trading activity. These preliminary results are perfectly consistent with Easley and O'Hara's (1992) predictions. Furthermore, the absolute value of a quote change after a large trade is large (see Hasbrouck (1991)). It is possible that these correlations simply reflect intraday deterministic patterns common to all the variables considered. Similar, although weaker, results are obtained, however, even after diurnally adjusting the series and, as we show in the next section, intraday periodicities have only a marginal role in explaining both quote changes and signed trade dynamics.

### III. Estimation and Results

In this section we study the effects of intertrade time durations on quote revision and autocorrelation of trades. Previous empirical analyses have shown that dynamics of trade durations (Engle and Russell (1998)), analogously to other market variables, can be attributed partly to intraday periodicities and partly to stochastic variations. Our first concern is then to use a set of dummy variables in the VAR estimation to distinguish between these two components. We consider nine diurnal dummy variables: one for the trades performed in the first 30 minutes after the open, then one for every hour of

<sup>15</sup> Lee and Ready (1991) suggest using the tick rule to reduce the number of unidentified trades, which means classifying a trade as a sale if the transaction price is lower than the previous transaction price or as a buy if the transaction price is higher than the previous transaction price. In the case of further uncertainty on the sign of the trade, this procedure can be iterated by comparing the current price with the price of earlier trades, moving further and further into the past.

the trading day until 3:00 p.m., and finally a 30 minute plus two 15 minute intervals during the last trading hour. We estimate the VAR defined by models (1), (5), and (6) with current and lagged values of all but one of the above dummy variables. As a starting point, we perform a Wald test of the null that all lagged diurnal dummies are jointly zero. Out of the 18 stocks in the sample, we reject the null for only two stocks in the quote revision equation (IBM and Exxon (XON) with  $p$ -values of 0.0463 and 0.0002, respectively) and for two other stocks in the trade equation (BA and T, with  $p$ -values of 0.0374 and 0.0001, respectively). The significance of  $\delta_i$  coefficients in such regressions is not weakened, but rather strengthened by the presence of diurnal dummies. Given this corroborating evidence, we exclude from the model past lags of diurnal dummies, focusing only on their current values. Reestimating the VAR, we do not recognize any prevailing daily pattern either for the price impact or for the autocorrelation of trades. Moreover, the estimates indicate that time effects in both equations can be predominantly attributed to the stochastic variation of trade durations  $T_t$ . Because most of the coefficients for the diurnal dummies are not significantly different from zero, we drop all but one of them. Only the trades performed in the first 30 minutes of the trading day tend to have a significantly different effect from the other trades. Therefore, we estimate the following system for quote revisions and trades

$$\begin{aligned}
 r_t &= \sum_{i=1}^5 a_i r_{t-i} + \lambda_{open}^r D_t x_t^0 + \sum_{i=0}^5 [\gamma_i^r + \delta_i^r \ln(T_{t-i})] x_{t-i}^0 + v_{1,t} \\
 x_t^0 &= \sum_{i=1}^5 c_i r_{t-i} + \lambda_{open}^x D_{t-1} x_{t-1}^0 + \sum_{i=1}^5 [\gamma_i^x + \delta_i^x \ln(T_{t-i})] x_{t-i}^0 + v_{2,t}. \quad (9)
 \end{aligned}$$

### A. The Relevance of Time in the Trade Equation

We consider first the effects of the time between trades on the autocorrelation of signed trades. That is, we investigate first own-variable effects before studying how time durations affect quote revisions. The bottom part of Table II contains the estimated coefficients for the trade equation in the case of a representative stock, Fannie Mae (FNM). The estimated coefficients for all the other stocks are in Table III. We format in bold the values of the coefficients that are significantly different from zero at the 5 percent level of confidence. Because heteroskedasticity is present in the residuals, we use White's heteroskedasticity consistent standard errors to compute Wald and  $t$ -statistics.

Results relative to the first two sets of coefficients have been discussed in Hasbrouck (1991). Signed trades exhibit strong positive autocorrelation. However, we are more concerned with the time coefficients,  $\delta$ s, which are typically negative (at least at the leading lags). Typically, the coefficient of the dummy variable for trades performed around the open is not significant. Thus, time effects in the trade equation are not attributable to daily varia-

**Table II**  
**Estimated Coefficients for Quote Revision and Trade Equation**  
**in the Fannie Mae (FNM) Case**

Coefficient estimates and *t*-statistics (in parentheses) for both quote revision and trade equations

$$r_t = \sum_{i=1}^5 a_i r_{t-i} + \lambda_{open}^r D_t x_t^0 + \sum_{i=1}^5 (\gamma_i^r + \delta_i^r \ln(T_{t-i})) x_{t-i}^0 + v_{1,t}$$

$$x_t^0 = \sum_{i=1}^5 c_i r_{t-i} + \lambda_{open}^x D_{t-1} x_{t-1}^0 + \sum_{i=1}^5 (\gamma_i^x + \delta_i^x \ln(T_{t-i})) x_{t-i}^0 + v_{2,t}$$

$r_t$  is the quote change after the trade in  $t$ ,  $x_{t-i}^0$  is the trade indicator (1 for a buy; -1 for a sale; 0 otherwise).  $T_t$  is the time between two consecutive transactions (+1 second).  $D_t$  is a dummy variable identifying trades performed in the first 30 minutes of the trading day. The *t*-statistics for the trade equation are computed using White's heteroskedasticity-consistent covariance estimator. The data reflect NYSE trading activity in FNM for 62 trading days from November 1990 through January 1991 (TORQ data). See Tables III and V for the results on the other stocks.

Lag Quote Revision	Lag Trade	Lag Trade Lag Duration	Lag Trade Diurnal Dummy	Adj. $R^2$
Panel A: Quote Revision Equation				
$a_1$	$\gamma_0$	$\delta_0$	$\lambda_{open}$	0.0629
<b>-0.0428</b> (-7.04)	<b>0.0259</b> (16.31)	<b>-0.0023</b> (-5.35)	<b>0.0052</b> (2.61)	
$a_2$	$\gamma_1$	$\delta_1$		
<b>-0.0144</b> (-2.35)	<b>0.0078</b> (4.94)	-0.0003 (-0.58)		
$a_3$	$\gamma_2$	$\delta_2$		
0.0047 (0.77)	<b>0.0073</b> (4.57)	<b>-0.0010</b> (-2.42)		
$a_4$	$\gamma_3$	$\delta_3$		
<b>0.0149</b> (2.45)	-0.0004 (-0.27)	0.0006 (1.39)		
$a_5$	$\gamma_4$	$\delta_4$		
<b>0.0179</b> (2.95)	0.0014 (0.87)	0.0000 (-0.02)		
	$\gamma_5$	$\delta_5$		
	0.0003 (0.19)	0.0002 (0.46)		
Panel B: Trade Equation				
$c_1$	$\gamma_1$	$\delta_1$	$\lambda_{open}$	0.1819
<b>-2.2713</b> (-1.81)	<b>0.3112</b> (19.97)	<b>-0.0155</b> (-3.64)	0.0021 (-0.11)	
$c_2$	$\gamma_2$	$\delta_2$		
<b>-1.1206</b> (-20.60)	<b>0.1670</b> (11.02)	-0.0064 (-1.55)		
$c_3$	$\gamma_3$	$\delta_3$		
<b>-0.4838</b> (-8.69)	<b>0.1056</b> (7.01)	-0.0067 (-1.64)		
$c_4$	$\gamma_4$	$\delta_4$		
<b>-0.2339</b> (-4.17)	<b>0.0619</b> (4.18)	0.0021 (0.52)		
$c_5$	$\gamma_5$	$\delta_5$		
0.1010 (1.78)	<b>0.0590</b> (4.09)	<b>-0.0088</b> (-2.21)		

Bold format denotes significance at the 5 percent level.

tions, but instead they seem to be primarily due to the stochastic component of time durations  $T_t$ . This interpretation is confirmed by the results of tests on time coefficients presented in Table IV. The first column of this table

**Table III**  
**Trade Equation in the Vector Autoregression**

Coefficient estimates and *t*-statistics for the trade equation.  $T_t$  is the time interval between two consecutive transactions;  $x_t^0$  is the trade indicator (1 if  $p_t > q_t$ ; -1 if  $p_t < q_t$ ; and 0 if  $p_t = q_t$ ), where  $p_t$  is the transaction price and  $q_t$  the midquote price).  $r_t$  is the percentage quote change after a trade.  $D_t$  is a dummy variable for trades around the open. The *t*-statistics (in parentheses) are computed using heteroskedasticity-consistent covariance estimators. The sample covers the period from November 1990 to January 1991 (TORQ data).

$$x_t^0 = \sum_{i=1}^5 c_i r_{t-i} + \lambda_{open} D_{t-1} x_{t-1}^0 + \sum_{i=1}^5 (\gamma_i + \delta_i \ln(T_{t-i})) x_{t-i}^0 + v_{2,t}$$

Stock	Lag Quote Revision					Lag Trade					Lag Trade * Lag Duration					Time $R^2$	No Time $R^2$	
	$c_1$	$c_2$	$c_3$	$c_4$	$c_5$	$\gamma_1$	$\gamma_2$	$\gamma_3$	$\gamma_4$	$\gamma_5$	$\delta_1$	$\delta_2$	$\delta_3$	$\delta_4$	$\delta_5$			$\lambda_{open}$
BA	-2.1360 (-34.72)	-1.1914 (-19.09)	-0.8339 (-12.89)	-0.3670 (-5.22)	-0.1201 (-1.86)	0.2323 (18.36)	0.1313 (10.72)	0.1116 (9.14)	0.0933 (7.71)	0.1008 (8.59)	0.0120 (-3.18)	-0.0061 (-1.66)	-0.0073 (-1.99)	-0.0039 (-1.07)	-0.0109 (-3.03)	0.0085 (0.51)	0.1171	0.1162
CAL	-0.1655 (-11.11)	-0.0970 (-6.38)	-0.0576 (-3.77)	-0.0242 (-1.57)	-0.0111 (-0.74)	0.2962 (7.36)	0.1760 (4.37)	0.1407 (3.48)	0.0746 (1.87)	0.0517 (1.34)	-0.0157 (-1.92)	-0.0087 (-1.08)	-0.0130 (-1.62)	0.0006 (0.08)	0.0034 (0.44)	-0.0004 (-0.01)	0.1238	0.1233
CL	-2.4266 (-19.36)	-0.445 (-4.45)	-0.1196 (-0.99)	-0.0254 (-0.22)	0.0879 (0.77)	0.3832 (13.40)	0.0916 (3.23)	0.0253 (0.88)	0.0435 (1.55)	0.0479 (1.84)	-0.0084 (-1.41)	-0.0045 (-0.76)	0.0075 (1.26)	-0.0021 (-0.35)	-0.0061 (-1.09)	0.0697 (1.78)	0.1592	0.1582
CPC	-2.0736 (-15.50)	-0.4481 (-3.62)	-0.1005 (-0.80)	-0.0454 (-0.37)	-0.1345 (-1.10)	0.3853 (10.51)	0.1567 (4.37)	0.0615 (1.70)	0.0368 (1.04)	0.0969 (2.90)	-0.0033 (-1.13)	-0.0216 (-3.01)	-0.0074 (-1.02)	-0.0017 (-0.24)	-0.0121 (-1.76)	-0.0242 (-0.46)	0.1448	0.1427
DI	-0.6710 (-13.54)	-0.3417 (-6.82)	-0.1799 (-3.66)	-0.1466 (-2.99)	-0.0675 (-1.43)	0.2939 (9.87)	0.1760 (6.16)	0.0678 (2.37)	0.0583 (2.05)	0.0444 (1.59)	-0.0091 (-1.42)	-0.0153 (-2.45)	-0.0018 (-0.30)	-0.0004 (-0.06)	-0.0030 (-0.49)	0.0269 (0.64)	0.1041	0.1032
FDX	-1.1171 (-16.07)	-0.3043 (-4.16)	-0.1229 (-1.81)	-0.0205 (-0.30)	0.0632 (0.96)	0.2614 (6.82)	0.1706 (4.95)	0.1004 (2.72)	0.0329 (0.94)	0.0333 (1.59)	0.0161 (2.05)	0.0063 (0.83)	-0.0052 (-0.69)	0.0017 (0.22)	-0.0019 (-0.26)	0.0351 (0.65)	0.1675	0.1666
FNM	-2.2713 (-18.99)	-1.206 (-20.60)	-0.4838 (-8.69)	-0.2539 (-4.17)	0.010 (1.78)	0.3112 (19.97)	0.1670 (7.00)	0.1056 (4.08)	0.0619 (1.82)	0.0990 (4.09)	-0.0155 (-3.64)	-0.0064 (-1.55)	-0.0067 (-1.64)	0.0021 (0.52)	-0.0088 (-2.21)	-0.0021 (-0.11)	0.1823	0.1815
FPL	-1.3392 (-13.99)	-0.9888 (-10.35)	-0.8979 (-9.26)	-0.5856 (-5.30)	-0.3567 (-3.54)	0.1532 (7.56)	0.1404 (6.92)	0.0828 (4.06)	0.0828 (4.14)	0.1182 (3.96)	0.0136 (2.67)	0.0023 (0.46)	0.0095 (1.87)	0.0063 (1.24)	-0.0020 (-0.41)	0.0373 (1.49)	0.0254	0.0245
GE	-3.3007 (-43.04)	-1.7921 (-22.27)	-1.0377 (-13.18)	-0.6522 (-8.30)	-0.3526 (-4.54)	0.2771 (31.73)	0.1460 (7.03)	0.0812 (3.52)	0.0777 (6.21)	0.0685 (8.34)	-0.0227 (-7.40)	-0.0087 (-2.87)	-0.0044 (-1.48)	-0.0075 (-2.52)	-0.0076 (-2.61)	0.0511 (4.42)	0.1260	0.1244
GLX	-1.6707 (-24.40)	-1.1117 (-15.62)	-0.5138 (-7.04)	-0.3959 (-5.44)	0.0237 (0.32)	0.2592 (13.93)	0.1316 (7.22)	0.0988 (6.45)	0.0787 (4.38)	0.0690 (5.08)	-0.0151 (-3.15)	-0.0017 (-0.35)	-0.0032 (-0.68)	-0.0008 (-0.17)	-0.0069 (-1.49)	0.0091 (0.39)	0.0951	0.0945
HAN	-1.0563 (-14.66)	-0.7185 (-10.18)	-0.4385 (-5.71)	-0.4059 (-5.27)	-0.0910 (-1.20)	0.2431 (6.26)	0.1608 (4.19)	0.1727 (4.52)	0.0938 (2.92)	0.0707 (1.90)	-0.0131 (-1.65)	-0.0152 (-2.09)	-0.0235 (-3.33)	0.0026 (0.33)	-0.0063 (-0.82)	0.0044 (0.10)	0.1009	0.0986
IBM	-6.6343 (-41.77)	-2.3648 (-18.64)	-0.8261 (-6.98)	-0.2032 (-1.73)	0.1841 (1.58)	0.3354 (35.95)	0.1412 (15.53)	0.0852 (9.55)	0.0468 (5.32)	0.0515 (6.13)	0.0009 (0.32)	-0.0017 (-0.59)	-0.0048 (-1.70)	-0.0028 (-1.02)	-0.0065 (-2.36)	0.0304 (2.56)	0.1852	0.1849
MO	-2.8712 (-38.04)	-2.0047 (-15.64)	-1.2543 (-15.06)	-0.9570 (-11.50)	-0.5642 (-6.70)	0.1690 (16.77)	0.1196 (12.22)	0.1023 (10.54)	0.0866 (8.99)	0.0636 (6.66)	-0.0149 (-4.39)	-0.0096 (-2.88)	-0.0060 (-1.80)	-0.0073 (-2.22)	-0.0040 (-1.21)	0.0238 (1.79)	0.0812	0.0804
NI	-8.8951 (-9.66)	-0.5246 (-6.40)	-0.4543 (-4.75)	-0.3609 (-3.89)	-0.1694 (-2.18)	0.4021 (10.78)	0.1285 (3.31)	0.0752 (3.31)	0.0528 (1.91)	0.0403 (4.03)	-0.0258 (-2.58)	0.0058 (1.05)	0.0063 (0.84)	0.0069 (0.95)	-0.0035 (-0.49)	0.0569 (1.14)	0.0668	0.0651
POM	-0.5219 (-5.67)	-0.4543 (-5.11)	-0.4973 (-5.60)	-0.389 (-3.89)	-0.150 (-1.50)	0.2301 (7.64)	0.1179 (3.95)	0.0545 (1.85)	0.0443 (1.50)	0.0406 (3.60)	-0.0047 (-0.73)	0.0028 (0.43)	0.0075 (1.16)	0.0107 (1.66)	-0.0021 (-0.33)	-0.0004 (-0.01)	-0.0238	-0.0244
SLB	-1.3272 (-16.94)	-0.4448 (-5.89)	-0.1079 (-1.43)	0.0124 (0.17)	0.0972 (1.27)	0.3244 (13.78)	0.1267 (5.48)	0.0503 (2.17)	0.0258 (1.13)	0.0571 (2.62)	-0.0130 (-2.31)	-0.0061 (-1.11)	0.0021 (0.39)	0.0021 (0.39)	-0.0107 (-2.03)	0.0398 (1.38)	0.1087	0.1078
T	-1.9386 (-27.70)	-1.3414 (-19.14)	-0.9900 (-14.08)	-0.7611 (-10.68)	-0.4495 (-6.20)	0.2252 (24.67)	0.1445 (16.07)	0.0893 (9.98)	0.1091 (12.29)	0.0903 (10.41)	-0.0177 (-5.55)	-0.0166 (-5.21)	-0.0036 (-1.14)	-0.0122 (-3.87)	-0.0088 (-2.85)	0.0692 (5.59)	0.1179	0.1179
XON	-2.6221 (-25.73)	-1.6158 (-15.97)	-0.9951 (-9.58)	-0.5732 (-5.38)	0.0340 (0.32)	0.2114 (12.51)	0.1005 (6.07)	0.1174 (7.19)	0.0918 (5.70)	0.0752 (4.74)	-0.0089 (-1.95)	-0.0089 (-1.50)	-0.0022 (-0.51)	-0.0034 (-0.77)	-0.0001 (-0.02)	0.0204 (0.99)	0.0429	0.0426

Bold format denotes significance at the 5 percent level.

**Table IV**  
**The Significance of Time in the Trade Equation**

Wald and *t* tests on the  $\lambda_{open}$  and  $\delta_i$  coefficients in the trade equation

$$x_t^0 = \sum_{i=1}^5 c_i r_{t-i} + \lambda_{open} D_{t-1} x_{t-1}^0 + \sum_{i=1}^5 (\gamma_i + \delta_i \ln(T_{t-i})) x_{t-i}^0 + v_{2,t}.$$

$r_t$  is the quote change after the trade in  $t$ ,  $x_{t-i}^0$  is the trade indicator (1 for a buy; -1 for a sale; 0 otherwise).  $T_t$  is the time between two consecutive transactions (+ 1 second).  $D_i$  is a dummy variable identifying trades performed in the first 30 minutes of the trading day. Wald and *t*-statistics are computed using White's heteroskedasticity-consistent covariance estimator. Data comprise NYSE trading activity for the period from November 1990 through January 1991 (TORQ data).

Stock	Diurnal and Stochastic Components	Diurnal Dummy	Stochastic Component	
	$\lambda_{open} = \delta_i = 0 \ (i = 1, \dots, 5)$	$\lambda_{open} = 0$	$\delta_i = 0 \ (i = 1, \dots, 5)$	$\sum \delta_i = 0$
$H_0:$	Wald Test	100 * $\lambda_{open}$	Wald Test	100 * $\sum \delta_i$
BA	<b>33.846</b>	0.848	<b>32.467</b>	<b>-4.0286</b>
CAL	8.771	-0.037	8.712	<b>-3.3355</b>
CL	8.981	6.974	5.411	-1.3619
CPC	<b>15.350</b>	-2.416	<b>15.254</b>	<b>-5.1191</b>
DI	9.267	2.693	8.595	-1.9924
FDX	5.792	3.513	5.560	1.6917
FNM	<b>26.677</b>	-0.208	<b>26.576</b>	<b>-3.5232</b>
FPL	<b>15.233</b>	3.732	<b>13.872</b>	<b>2.9681</b>
GE	<b>120.934</b>	<b>5.108</b>	<b>88.444</b>	<b>-5.0962</b>
GLX	<b>15.321</b>	0.911	<b>14.147</b>	<b>-2.7636</b>
HAN	<b>18.632</b>	0.438	<b>18.105</b>	<b>-5.5525</b>
IBM	<b>18.993</b>	<b>3.042</b>	<b>11.238</b>	<b>-1.4861</b>
MO	<b>50.725</b>	2.376	<b>43.065</b>	<b>-4.1709</b>
NI	9.816	5.691	8.517	-0.1250
POM	4.952	-0.041	4.948	1.4127
SLB	<b>13.627</b>	3.979	<b>11.088</b>	<b>-2.5540</b>
T	<b>152.508</b>	<b>6.925</b>	<b>105.614</b>	<b>-5.8919</b>
XON	7.941	2.038	6.620	-0.7804

Bold format denotes significance at the 5 percent level.

shows Wald statistics for the null hypothesis that time coefficients are jointly zero. The null hypothesis is rejected for 11 out of the 18 stocks in the sample. This is particularly evident for T and GE, which are the most frequently traded stocks of our sample. However, the coefficient of the dummy for trades performed around the open is significantly different from zero in only three cases. In the last two columns of Table IV we show, respectively, Wald statistics for the null hypothesis that all the coefficients for the stochastic component of time,  $\delta_i$ , are jointly zero, and that the sums of these coefficients are zero. The sums of the  $\delta_i$  coefficients are negative for 16 stocks and are significantly different from zero for 11 stocks. Thus summarizing, high trading activity induces stronger positive autocorrelation of signed trades.

*B. The Relevance of Time in the Quote Revision Equation*

We then turn to study the relevance of time effects in the first equation of the system. The estimated coefficients and  $t$ -statistics for the quote revision equation of system (9) for FNM are presented in the top part of Table II. Similar results for all stocks are in Table V. The coefficients on lagged price changes are generally negative at the first lag, indicating negative correlation in returns. The most important sets of coefficients for our investigation are the  $\gamma_i$ s and  $\delta_i$ s, which are, respectively, the coefficients on the trade indicator and the interaction of trade indicator and duration. These coefficients reflect the price impact of a signed trade. By observing the estimates for these coefficients, we notice that the price impact of a signed trade is positive, generally persistent, and negatively related to durations. The  $\gamma_{open}$  coefficient corrects for any anomalous price effects resulting from opening trades. Although for Fannie Mae there is evidence of higher price impact of trades around the open, this is not a general result (see the second column of Table VI).

The coefficients on the interaction between the trade indicator and the time duration are negative and significantly different from zero at the first lag for 13 out of the 18 sample stocks. *A buy transaction arriving after a long time interval has a lower price impact than a buy transaction arriving right after a previous trade.* The interpretation is that the market maker infers a higher likelihood of informed traders if the trades are close together. The presence of informed traders may also deter the uninformed from trading, further increasing the proportion of informed trading. Therefore, it is more difficult to find liquidity suppliers willing to take the opposite side of a transaction, and so trades have a higher impact on prices. To assess the significance of the role played by time, we perform a Wald test of the null hypothesis that  $\delta_i$  coefficients are jointly zero. The results of the Wald test are presented in the fourth column of Table VI. We reject the null for 13 stocks. To determine whether time affects only short term price variations or also long term price adjustments to information, we perform a Wald test of the null that the sum of  $\delta_i$  coefficients is zero. We reject the null with 5 percent level of confidence for 13 out of 18 stocks and the sum is negative for 17 of these (see Table VI).

These equations predict returns and trade directions on a trade-by-trade basis. Naturally, the  $R^2$  of these regressions are quite low. When time is added to the model, the fit may be significantly better on statistical grounds, but still reflects only minimal predictability. For instance, if we compare the  $R^2$  (see Tables III and VIII) for the trade and the return equations with and without the time coefficients, we observe only small enhancements to the  $R^2$  of the regressions. Nevertheless, because we are studying the dynamics of prices at the transaction level, even an imperceptible systematic improvement on the per share price paid (which in this case, as we showed, is statistically significant) could translate into substantial effects. This will become more apparent when the impulse responses are computed.



**Table VI**  
**The Significance of Time in the Return Equation**

Wald and  $t$  tests on the  $\lambda_{open}$  and  $\delta_i$  coefficients in the quote revision equation

$$r_t = \sum_{i=1}^5 \alpha_i r_{t-i} + \lambda_{open} D_t x_t^0 + \sum_{i=0}^5 (\gamma_i + \delta_i \ln(T_{t-i})) x_{t-i}^0 + v_{1,t}.$$

$r_t$  is the quote change after the trade in  $t$ ,  $x_{t-i}^0$  is the trade indicator (1 for a buy; -1 for a sale; 0 otherwise).  $T_t$  is the time between two consecutive transactions (+1 second).  $D_t$  is a dummy variable identifying trades performed in the first 30 minutes of the trading day. Data comprise NYSE trading activity for the period from November 1990 through January 1991 (TORQ data).

Stock	Diurnal and Stochastic Components	Diurnal Dummy	Stochastic Component	
	$\lambda_{open} = \delta_i = 0 (i = 1, \dots, 5)$	$\lambda_{open} = 0$	$\delta_i = 0 (i = 1, \dots, 5)$	$\Sigma \delta_i = 0$
H <sub>0</sub> :	Wald Test	100 * $\lambda_{open}$	Wald Test	100 * $\Sigma \delta_i$
BA	<b>22.884</b>	0.203	<b>18.531</b>	<b>-0.2184</b>
CAL	11.148	6.056	9.110	<b>-2.8622</b>
CL	<b>128.663</b>	0.413	<b>124.217</b>	<b>-1.1140</b>
CPC	<b>42.806</b>	0.746	<b>40.009</b>	<b>-0.6772</b>
DI	<b>19.247</b>	<b>2.020</b>	<b>12.779</b>	-0.5362
FDX	<b>43.436</b>	1.047	<b>40.944</b>	<b>-1.2901</b>
FNM	<b>47.626</b>	<b>0.524</b>	<b>37.922</b>	<b>-0.2821</b>
FPL	<b>24.603</b>	<b>0.508</b>	<b>13.985</b>	<b>-0.2410</b>
GE	<b>15.017</b>	<b>0.129</b>	8.777	-0.0025
GLX	8.040	0.207	6.374	-0.1040
HAN	<b>26.453</b>	0.154	<b>26.065</b>	-0.3180
IBM	<b>257.475</b>	<b>-0.244</b>	<b>244.399</b>	<b>-0.2301</b>
MO	<b>20.642</b>	<b>-0.139</b>	<b>17.256</b>	<b>-0.1158</b>
NI	6.292	-1.048	5.135	-0.1858
POM	10.827	0.573	8.595	<b>-0.4050</b>
SLB	<b>63.653</b>	<b>1.235</b>	<b>44.165</b>	<b>-0.6928</b>
T	<b>20.558</b>	0.103	<b>18.273</b>	<b>0.1091</b>
XON	<b>23.416</b>	0.028	<b>23.011</b>	<b>-0.1118</b>

Bold format denotes significance at the 5 percent level.

### C. Analysis of the System's Dynamics

In the analysis of the system's dynamics we use the bivariate VAR for quote changes and trades as defined in model (9) and we assume time durations are strongly exogenous. By computing the impulse response function for quote revision  $I_r(\cdot)$ , we can determine the expected long-run price impact of an unexpected trade (i.e., of a shock to the trade equation).

We assume that the trade shock disturbs an equilibrium phase of the system for prices and trades. We define a steady-state equilibrium to be when traders trade at the midquote (i.e.,  $x_t^0 = 0$ ) and the specialist does not change quotes ( $r_t = 0$ ). All future returns are then expected to be zero in equilibrium.  $I_r(\cdot)$  is defined as the conditional expectation of  $r_t$ ,  $k$  steps into the

future, after a trade shock hits the system in  $t$ , and the  $t$ th trade has a time duration of  $\tau$ . If we define  $\omega_{t-1}$  as the specific history of the variables in the model up to time  $t - 1$ , then we can write

$$I_r(k, v_t, T_t, \omega_{t-1}) = E[r_{t+k} | v_{2,t} = 1, T_t = \tau, \omega_{t-1}], \quad (10)$$

where we integrate out all other contemporaneous and future shocks. In particular, to calculate the impulse response, we must recognize that we have modeled prices conditional on durations. Thus, the joint density of prices and durations is needed to compute impulse responses. This, however, is simply the product of the conditional density of prices times the marginal density of durations. Here we use the ACD model to represent the future stochastic paths of durations. Furthermore, because  $I_r(\cdot)$  is a nonlinear function of time durations, we adopt a similar methodology to the one introduced by Koop, Pesaran, and Potter (1996) to numerically integrate out time durations via a Monte Carlo experiment,

$$I(\cdot) = E_T[E(r, x^0 | T)].$$

Once we average the impulse responses over all possible time sequences that originate from given initial conditions  $\omega_{t-1}$  and  $T_t = \tau$ , then the computation becomes straightforward as in the case of a linear system with diagonal variance-covariance matrix of disturbances.

We now consider a limiting case. Specifically, we compare the price impact of an average buy trade when the market activity is very low with the impact of an average buy trade when the market activity is high. To this purpose, we select from the sample a trade performed around 12:30 p.m. on December 24, 1990, and a trade performed around 10:00 a.m. on January 17, 1991. These are the days with, respectively, the lowest (highest) number of transactions and highest (lowest) average durations. In addition, 12:30 lies in the intraday period with the lowest trading intensity (because of the lunch effect), whereas 10:00 lies in the intraday period with the highest trading intensity. The histories, which correspond to these two trades, are used as initial conditions for the computation of the impulse responses. Specifically, to model and simulate the trading rate, we proceed through the following steps.

1. We filter out the time-of-the-day effect (deterministic component) from the duration series by fitting a piece-wise linear spline. Hence, we divide the original data by the deterministic component to obtain a diurnally adjusted series,  $\bar{T}_t$ , with unit mean.
2. We fit the Weibull ACD model (W-ACD) on the adjusted durations  $\bar{T}_t$ , and estimate the series of conditional mean durations  $\phi_t \Gamma(1 + 1/\theta)$ . The estimated coefficients for a W-ACD(2,1) model fitted on time durations for Fannie Mae and IBM are presented in Table VII.

**Table VII**  
**W-ACD estimation for FNM and IBM**

We estimate the Weibull Autoregressive Conditional Duration (W-ACD) model on FNM and IBM intertrade durations (+1 second) after removing the time-of-the-day effect through a piecewise linear spline. We use duration data for the period from November 1990 through January 1991 (TORQ data).  $\Phi_t(t-1)$  is the seasonal component of intertrade arrival times, and depends on trade's time up to  $t-1$ . The deseasonalized duration  $\tilde{T}$  is defined as  $\tilde{T}_t \equiv T_t/\Phi_t(t-1)$ . Assuming that  $\tilde{T}$  has a Weibull distribution, the estimated W-ACD(2,1) model is

$$\phi_t = \omega + \rho_1 \tilde{T}_{t-1} + \rho_2 \tilde{T}_{t-2} + \zeta \phi_{t-1} + \lambda D_{t-1}$$

and

$$g(\tilde{T}_t) = \frac{\theta}{\phi_t^\theta} \tilde{T}_t^{\theta-1} \exp \left[ - \left( \frac{\tilde{T}_t}{\phi_t} \right)^\theta \right] \quad \text{for } \theta, \phi_t > 0.$$

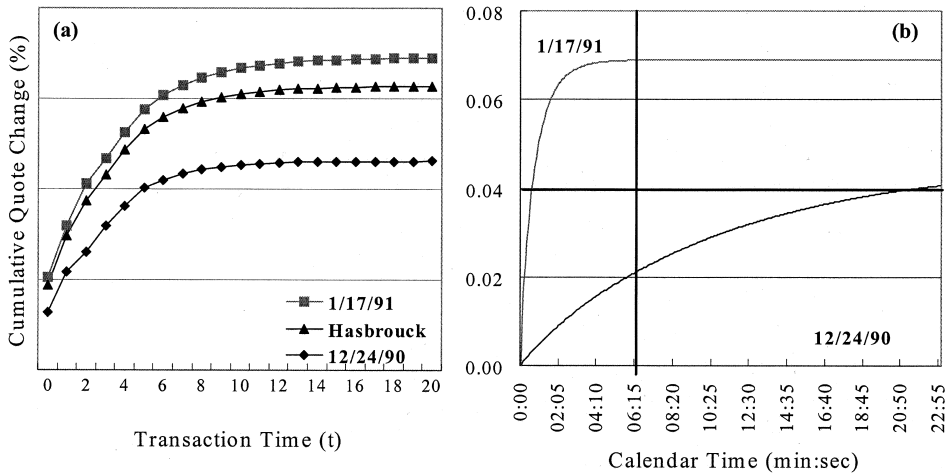
$D_t$  is a dummy variable for the first observation of the trading day.

	FNM	IBM
	Coefficient ( <i>t</i> -statistic)	Coefficient ( <i>t</i> -statistic)
$\omega$	<b>0.0072</b> <b>(8.61)</b>	<b>0.0048</b> <b>(10.57)</b>
$\rho_1$	<b>0.1262</b> <b>(18.60)</b>	<b>0.0804</b> <b>(18.56)</b>
$\rho_2$	<b>-0.0803</b> <b>(-11.58)</b>	<b>-0.0334</b> <b>(-7.52)</b>
$\zeta$	<b>0.9445</b> <b>(367.18)</b>	<b>0.9457</b> <b>(566.60)</b>
$\theta$	<b>0.8962</b> <b>(213.10)</b>	<b>0.8845</b> <b>(292.96)</b>
$D_{new\ day}$	<b>-0.0912</b> <b>(-3.12)</b>	<b>-0.1806</b> <b>(-6.84)</b>
$\Gamma(1 + 1/\theta)^a$	1.055	1.062
<i>Likelihood</i>	-26943.6	-52718.4

<sup>a</sup>The condition for stationarity  $\Gamma(1 + 1/\theta) * (\rho_1 + \rho_2) + \zeta < 1$ , where  $\Gamma(\cdot)$  is the gamma function, is satisfied for both stocks.

Bold format denotes significance at the 5 percent level.

3. We simulate the W-ACD for  $k$  steps, given initial conditions  $\omega_{t-1}$  and  $\tau$ , and we put back the deterministic component for each realization. In our particular case, the initial conditions for the simulation are simply the current and lag duration  $T_t$ ,  $T_{t-1}$  and  $\phi_t$ .
4. We compute the impulse response function for the  $k$  steps into the future. This is a realization of the random variable  $I_r(\cdot)$  for a given time path. At the same time, we tabulate prices as a function of calendar time.

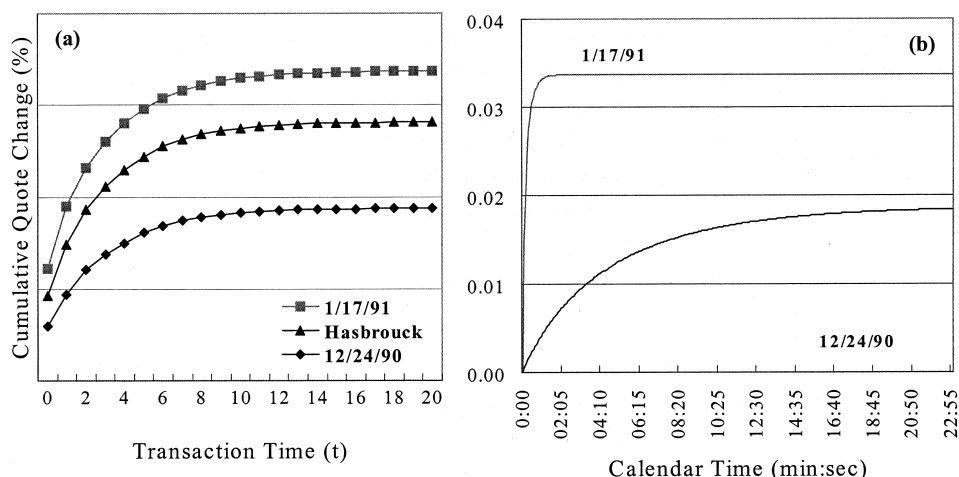


**Figure 1. Impulse Response Function for the FNM stock.** Cumulative impulse response functions for the percentage quote revision after an unexpected buy. The impulse responses are computed for two different initial conditions on time duration. We use the time duration and the conditional duration for a trade that occurred around 10:00 a.m. on January 17, 1991, and again the time duration and the conditional duration for a trade that occurred around 12:30 p.m. on December 24, 1990. For panel (a), we compute 10,000 impulse response functions for 21 steps into the future. Then, we compute the average of the cumulative quote change at every step. The impulse response for a model with time durations' effects ignored (Hasbrouck (1991)) is also charted. For panel (b), we use the methodology described in Section III.C.; that is, we compute 201 steps for 10,000 responses. Then, we sample every 5 seconds from the 10,000 impulse responses, and we compute the average cumulative quote revision.

5. We repeat (3) and (4) 10,000 times and then average the 10,000 conditional expectations at every step  $k$ , obtaining an estimator of the impulse response function in trade time.

The cumulative percentage price impact of an unexpected trade performed in both cases of high and low trading intensity with  $k = 21$  is presented in Figure 1a (for Fannie Mae) and Figure 2a (for IBM). These two scenarios (the most active time on the most active day versus the least active time on the least active day) are useful in that they establish a range of what we are likely to encounter. As expected, the impulse-response function for the VAR specification with time ignored (Hasbrouck (1991)) lies in between the other two. These impulse responses are plotted in "event time"; that is, we measure the evolution of prices as the number of transactions grows. When trading intensity is higher (short durations), the long-run price impact is also higher.

Furthermore, with the ACD model, we can compute the percentage price variation following an unexpected buy not only in event time, but also in calendar time (see Figures 1b and 2b). To generate the impulse responses in calendar time, we repeat steps (3) and (4) in the algorithm above, but this



**Figure 2. Impulse Response Function for the IBM stock.** Cumulative impulse response functions for the percentage quote revision after an unexpected buy. The impulse responses are computed for two different initial conditions on time duration. We use the time duration and the conditional duration for a trade that occurred around 10:00 a.m. on January 17, 1991, and again the time duration and the conditional duration for a trade that occurred around 12:30 p.m. on December 24, 1990. For panel (a), we compute 10,000 impulse response functions for 21 steps into the future. Then, we compute the average of the cumulative quote change at every step. The impulse response for a model with time durations' effects ignored (Hasbrouck (1991)) is also charted. For panel (b), we use the methodology described in Section III.C.; that is, we compute 201 steps for 10,000 responses. Then, we sample every 5 seconds from the 10,000 impulse responses, and we compute the average cumulative quote revision.

time we compute longer impulse response functions (201 steps) for 10,000 times. Hence, we sample the cumulative price changes every five seconds and we take averages. We consider, as an example, the case of the Fannie Mae stock (see Figure 1b). If an unexpected buy occurs at 10:00 a.m. on January 17, 1991, the cumulative percentage price variation, exactly 6 minutes and 15 seconds after the trade, will be 0.0689 percentage points. This is more than three times the cumulative price variation that follows an unexpected buy at 12:15 p.m. on December 24, 1990.

The inclusion of duration in the system makes it possible to solve three different types of problems. First, by drawing a vertical line on the graph in Figure 1b, we can answer the question, "How much will price change after an unexpected buy in a given period of time?" Second, by drawing a horizontal line on the graph in Figure 1b, we can determine how long it will take until prices change by a given amount. For example, we can compute the time it takes to change prices by 0.04 percentage points, which will be different at any moment of the trading day. Last, by defining a sufficiently small value,  $\epsilon > 0$ , such that every percentage price change lower than  $\epsilon$  is considered zero, we can compute the time it takes for prices to converge to the full information level after a trade. In Figure 1b, we show that prices

converge to the full information level in about four minutes when trading intensity is high, whereas it takes more than 23 minutes when trading intensity is low.

When trading activity is high, the convergence rate of prices to the full information value after an unexpected trade is faster. This supports the picture of a market with a higher proportion of informed traders, which consequently shows faster adjustment of prices to information and greater sensitivity of prices to trades.

This section clearly points out the general applicability of the ACD model to event-time market microstructure results. The ACD model can be employed to shift from event time to calendar time, and gain further useful information on the process of price dynamics. Even if time durations were not explicitly incorporated into the model for prices and trades, the ACD model could still be used to show that in real time, prices respond more quickly when the market is fast than when it is slow.

#### D. Robustness of Results for Time

At this point we know that time *per se* is informative, yet time may not offer any new information besides that conveyed by other variables such as volume and spread. By adding these variables to the specification of both the trade impact and autocorrelation of trades we have

$$(b_i, d_i) = \gamma_i + \delta_{1,i} \ln(T_{t-i}) + \delta_{2,i} \ln(\text{SIZE}_{t-i}/100) + \delta_{3,i} \text{SPREAD}_{t-i}. \quad (11)$$

*SIZE* is the trade size, and *SPREAD* is the difference between ask and bid price. *SIZE* enters the price impact specification nonlinearly as in Hasbrouck (1991).

The use of spread and volume in the trade impact parameterization is consistent with the theory of asymmetric information models and with previous empirical results. When the market maker observes large trades, s/he fears the possibility of losses by trading with the informed and widens the spread.

When we add spread and volume to time duration as determinants of the trade impact on quote revision, time duration still enters significantly for eight stocks and with negative sign (at least at the first lag) for 15 stocks. We perform two tests on each group of coefficients related to every single explanatory variable in the trade impact specification (11). For every stock in the sample we test the null hypothesis that all the coefficients in each group are jointly zero and after computing their sum we also test that the sum is zero.

The sum of the coefficients for each group provides a first raw approximation of the long-run impact on prices of any specific explanatory variable. A summary of the results for the quote revision equation is presented in Table VIII. Time duration enters significantly for 8 stocks out of 18 in the most general specification and for 6 of these also preserves its negative sign.

**Table VIII**  
**Robustness of Results on Time for the Quote Revision Equation**

This table presents tests on the coefficients of the quote revision equation

$$r_t = \sum_{i=1}^5 a_i r_{t-i} + \lambda_{open} D_t + \sum_{i=0}^5 b_i x_{t-i}^0 + v_{1,t},$$

where the trade impact on quotes,  $b_i$ , is parameterized as

$$b_i = \gamma_i + \delta_{1,i} \ln(T_{t-i}) + \delta_{2,i} \ln(SIZE_{t-i}/100) + \delta_{3,i} SPREAD_{t-i}.$$

$r_t$  is the quote change after the trade in  $t$ ,  $x_{t-i}^0$  is the trade indicator (1 for a buy; -1 for a sale; 0 otherwise).  $T_t$  is the time between two consecutive transactions (+ 1 second).  $SIZE$  is the number of transacted shares.  $SPREAD$  is the difference between ask and bid price.  $D_t$  is a dummy variable identifying trades performed in the first 30 minutes of the trading day. The sample covers the period from November 1990 to January 1991 (TORQ data). We consider the group of coefficients for each regressor and we test the null hypothesis that these  $\delta_i$  coefficients are jointly zero. We also compute the sum of the coefficients in each group and test the null hypothesis that this sum is zero. The tail probabilities for the Wald statistics are presented. At the bottom of the table we summarize the number of stocks for which the null hypothesis is rejected and the prevailing sign for the sum of the coefficients (e.g., (+) 18/18 means that the positive sign prevailed for all stocks).

Stock	Time Duration		Volume		Spread		
	$H_0: \delta_{1,i}$ jointly null		$H_0: \delta_{2,i}$ jointly null		$H_0: \delta_{3,i}$ jointly null		
	Prob > $\chi^2$	$100 * \Sigma \delta_{1,i}$	Prob > $\chi^2$	$\Sigma \delta_{2,i}$	Prob > $\chi^2$	$\Sigma \delta_{3,i}$	
BA	0.877	-0.033	0.578	0.013	0.091	0.000	
CAL	0.376	-2.044	0.129	0.173	2.862	0.000	
CL	0.000	-0.880	0.000	0.020	0.068	0.001	
CPC	0.000	-0.589	0.000	0.017	0.028	0.197	
DI	0.452	0.079	0.782	0.035	0.184	0.000	
FDX	0.000	-1.254	0.000	0.024	-0.106	0.009	
FNM	0.259	0.047	0.618	0.017	0.258	0.000	
FPL	0.443	-0.082	0.271	0.013	0.312	0.000	
GE	0.000	0.167	0.000	0.010	0.133	0.000	
GLX	0.874	-0.018	0.835	0.013	0.085	0.000	
HAN	0.014	0.019	0.940	0.034	0.531	0.000	
IBM	0.000	-0.046	0.047	0.007	0.074	0.000	
MO	0.978	0.004	0.896	0.007	0.085	0.000	
NL	0.856	-0.056	0.774	0.022	0.068	0.080	
POM	0.537	-0.259	0.079	0.032	0.433	0.000	
SLB	0.000	-0.542	0.000	0.015	0.008	0.587	
T	0.000	0.155	0.000	0.006	0.092	0.000	
XON	0.120	-0.001	0.979	0.009	0.091	0.000	
Summary	8/18	(-) 12/18	7/18	(+) 18/18	18/18	(+) 17/18	15/18

Bold format denotes significance at the 5 percent level.

The evidence suggests that dynamics of volume and spread predominantly characterize the price impact of trades and, especially for some stocks, the net effect of time duration is only marginal. These results for transaction data seem to differ from what Jones, Kaul, and Lipson (1994) find for daily data. At the aggregate daily level, the number of trades rather than the average trade size is the component of aggregate volume that best explains daily price volatility.

In the trade equation, time seems to be slightly more robust to the presence of volume and spread than for the quote revision equation. In fact, time duration is still significant for 10 stocks, 9 of which also have the expected negative sign (we do not present the results for the trade equation, but these are available from the authors upon request).<sup>16</sup>

### *E. Exogeneity of Trade Arrival Process*

At least two important asymmetric information models (Glosten and Milgrom (1985) and Easley and O'Hara (1987)) resort to exogeneity of the trade arrival process when dealing with trades' timing in models for prices and trades. The exogeneity of time duration, which in our paper becomes strong exogeneity (in the sense of Engle et al. (1983)), is used as a simplifying operative assumption. In our model in particular, it is required for the computation of impulse response functions (see Section III.C) and it implies that the information set for forecasting the next transaction arrival does not include past prices and trades. However, this assumption may be too restrictive. Starting with the pioneering work of Garman (1976), many contributions to the market microstructure literature conjecture, in fact, that the trade arrival rate might depend on market variables such as prices. Engle and Russell (1994) offer some preliminary empirical evidence for IBM data. Easley and O'Hara (1992) theorize correlations between intertrade times and prices, spread, and volume. Yet, their model formalizes only the effects of time on the other variables. We do not have knowledge, to this date, of any theoretical model that shapes the reciprocal interactions of price, trade, and time, thus also offering an explanation for any causality running in the opposite direction. In particular, it might be interesting to study the effects, if any, of market maker's quote revision on the trading flow. A scenario consistent with asymmetric information models would be that the quote change chokes off the trading flow in the absence of an information event, thus leading the

<sup>16</sup> The model can also be used to study the impact that specific trades have on prices. For example, we add to the trade impact parameterization  $b_i$ , dummy variables that identify off-exchange, midquote, and buy trades. Our results show that trades performed on regional exchanges and Nasdaq significantly affect NYSE quotes, but have a relatively lower impact, that is, are less informative, than trades performed on NYSE (see Hasbrouck (1995) and Blume and Goldstein (1997) for further evidence). Midpoint trades, classified through the tick test (Lee and Ready (1991)) do not cause a unidirectional change in quotes and have a smaller price effect. This result reinforces the strategy adopted in the paper of assigning a zero weight to price effects of midquote trades. Finally, we do not find conclusive evidence of asymmetric price impact of buyer- and seller-initiated transactions.

market back to a “normal” trading level. From an inventory-model perspective instead, quote change would immediately attract opposite side traders (see O’Hara (1995)).

We therefore investigate if the residuals of the ACD estimation contain any evidence supporting these conjectures. We first diurnally adjust the duration series for every stock with a piece-wise linear spline.<sup>17</sup> Second, we estimate W-ACD(2,1) models on the adjusted duration series.<sup>18</sup> We stated in Section I.C that a particular transformation of the ACD residuals gives the so-called standardized durations  $\epsilon_t \equiv (\tilde{T}_t/\phi_t)^\theta$ , which under the null of correct specification are i.i.d. exponential(1). We then run the following regression

$$\epsilon_t = m + \sum_{i=1}^5 a_i |\tilde{r}_{t-i}| + \sum_{i=1}^5 b_i D_{t-i}^x + \sum_{i=1}^5 c_i \epsilon_{t-i} + \sum_{i=1}^5 d_i \tilde{x}_{t-i} + \sum_{i=1}^5 e_i \tilde{s}_{t-i} + v_t, \quad (12)$$

where  $m$ ,  $a_i$ ,  $b_i$ ,  $c_i$ ,  $d_i$  and  $e_i$ ,  $i = 1, \dots, 5$  are all parameters. This regression is used to test for omitted variables in the specification of the conditional mean of the ACD model. We take five lags to account for persistent effects. We use the following regressors: diurnally adjusted absolute value of returns,  $|\tilde{r}_t|$ , diurnally adjusted  $\ln(\text{SIZE}_t/100)$ ,  $\tilde{x}_t$ , diurnally adjusted spread (ask price minus bid price),  $\tilde{s}_t$ , a trade dummy variable,  $D_t^x$ , which takes a value of one for trades with transaction price different from the midquote and, finally, past standardized durations,  $\epsilon_t$ . The presence of a daily periodic component in most of the high-frequency variables is well documented (see footnote 8). We eliminate this component from absolute return, log size, and spread by applying the same methodology used for durations. We also include in the regression past standardized durations because, even though the W-ACD model captures most of the autoregressive structure, very often we cannot accept the null hypothesis of zero autocorrelation of residuals. This test also has power to detect inadequacies in the dynamic specifications of the ACD.

$P$ -values for Wald tests of the null hypothesis that all of the coefficients related to every single regressor are jointly zero and that the sums of these coefficients are zero are presented in Table IX. The sum of the coefficients is a robust indicator for the direction of the effect of each regressor on durations. In Table IX, we format in bold stocks for which the null hypothesis that the coefficients for each group are jointly zero is rejected at 5 percent

<sup>17</sup> In particular, we define a piece-wise linear spline with eight nodes. The nodes are fixed at 9:30 a.m. (the official opening time), 10:00, 11:00, 12:00, 1:00 p.m., 2:00, 3:00, 3:30, and 4:00 (the official closing time). That is, we use a linear approximation for every hour period except for the opening and the closing, where, due to higher trading intensity, we break the time intervals in 30 minute spans.

<sup>18</sup> As noted previously, we deal with zero durations by augmenting them by one second before eliminating the periodic component.

Table IX

Standardized Duration Equation

We first diurnally adjust the duration series with a piece-wise linear spline. Second, we estimate W-ACD(2,1) models for the adjusted duration series. The sample covers the period from November 1990 to January 1991 (TORQ data). A particular transformation of the ACD residuals gives the standardized durations, which under the null of correct specification for the ACD's conditional mean are i.i.d.  $\exp(1)$ . We then run the regression

$$\epsilon_t = m + \sum_{i=1}^5 a_i |\bar{r}_{t-i}| + \sum_{i=1}^5 b_i D_{t-i}^x + \sum_{i=1}^5 c_i \epsilon_{t-i} + \sum_{i=1}^5 d_i \bar{x}_{t-i} + \sum_{i=1}^5 e_i \bar{s}_{t-i} + v_t,$$

where  $\epsilon_t$  is standardized duration,  $|\bar{r}_t|$  is absolute value of diurnally adjusted return,  $D_t^x$  is a dummy variable for trades not at the midquote,  $\bar{x}_t$  is diurnally adjusted  $\ln(\text{SIZE}_{t-i}/100)$ , and  $\bar{s}_t$  is diurnally adjusted spread. Absolute return, log size, and spread series are adjusted with the same method used for durations. Spread is defined as the difference between ask and bid price.

Stock	Return		Trades not at Midquote		Standardized Duration		Volume		Spread		Regression	
	$H_0: a_i$		$H_0: b_i$		$H_0: c_i$		$H_0: d_i$		$H_0: e_i$		All 25	
	jointly = 0	$\Sigma a_i$	jointly = 0	$\Sigma b_i$	jointly = 0	$\Sigma c_i$	jointly = 0	$\Sigma d_i$	jointly = 0	$\Sigma e_i$	coefficients = 0	$\text{Prob} > \chi^2$
BA	<b>0.000</b>	<b>-0.027</b>	<b>0.008</b>	<b>-0.088</b>	<b>0.000</b>	<b>0.063</b>	<b>0.000</b>	<b>-0.075</b>	0.424	0.024	<b>0.000</b>	0.008
CAL	0.419	0.002	0.891	0.098	<b>0.009</b>	<b>0.080</b>	<b>0.001</b>	<b>-0.166</b>	0.136	0.038	<b>0.000</b>	0.012
CL	<b>0.000</b>	<b>-0.046</b>	<b>0.000</b>	<b>-0.167</b>	0.090	0.051	<b>0.000</b>	<b>-0.027</b>	<b>0.011</b>	<b>0.071</b>	<b>0.000</b>	0.013
CPC	<b>0.022</b>	<b>-0.046</b>	0.100	-0.037	0.110	0.061	<b>0.039</b>	<b>-0.054</b>	<b>0.000</b>	<b>0.132</b>	<b>0.000</b>	0.010
DI	<b>0.012</b>	<b>-0.020</b>	<b>0.049</b>	<b>-0.039</b>	0.346	0.028	<b>0.005</b>	<b>-0.068</b>	0.281	0.035	<b>0.000</b>	0.006
FDX	<b>0.000</b>	<b>-0.034</b>	<b>0.003</b>	<b>-0.101</b>	<b>0.000</b>	<b>0.117</b>	0.134	<b>-0.052</b>	<b>0.004</b>	<b>0.097</b>	<b>0.000</b>	0.014
FNM	0.269	0.008	<b>0.000</b>	<b>-0.180</b>	<b>0.000</b>	<b>0.056</b>	<b>0.000</b>	<b>-0.191</b>	<b>0.026</b>	<b>-0.125</b>	<b>0.000</b>	0.010
FPL	0.635	0.000	<b>0.009</b>	<b>-0.186</b>	0.624	0.006	<b>0.000</b>	<b>-0.120</b>	0.443	<b>-0.045</b>	<b>0.000</b>	0.005
GE	0.359	-0.005	0.918	-0.008	0.642	0.006	0.766	-0.011	0.987	-0.016	0.896	0.001
GLX	0.129	-0.005	0.259	-0.071	<b>0.006</b>	<b>0.040</b>	<b>0.000</b>	<b>-0.061</b>	0.574	0.019	<b>0.000</b>	0.003
HAN	0.434	0.000	0.272	-0.129	<b>0.008</b>	<b>0.068</b>	<b>0.000</b>	<b>-0.123</b>	0.812	0.009	<b>0.000</b>	0.010
IBM	<b>0.000</b>	<b>0.020</b>	<b>0.000</b>	<b>-0.238</b>	<b>0.000</b>	<b>0.011</b>	<b>0.000</b>	<b>-0.147</b>	<b>0.000</b>	<b>-0.091</b>	<b>0.000</b>	0.010
MO	<b>0.000</b>	<b>-0.005</b>	<b>0.020</b>	<b>-0.083</b>	<b>0.000</b>	<b>0.029</b>	<b>0.000</b>	<b>-0.121</b>	<b>0.000</b>	<b>-0.077</b>	<b>0.000</b>	0.007
NI	0.098	-0.011	0.986	0.024	<b>0.000</b>	<b>0.137</b>	0.776	-0.024	0.068	-0.027	<b>0.000</b>	0.010
POM	<b>0.009</b>	<b>-0.015</b>	0.064	-0.084	<b>0.016</b>	<b>0.079</b>	<b>0.001</b>	<b>-0.049</b>	0.349	0.058	<b>0.000</b>	0.008
SLB	<b>0.000</b>	<b>-0.047</b>	<b>0.016</b>	<b>-0.084</b>	<b>0.000</b>	<b>0.075</b>	<b>0.000</b>	<b>-0.049</b>	<b>0.000</b>	<b>0.086</b>	<b>0.000</b>	0.008
T	0.973	0.001	0.774	0.001	<b>0.001</b>	<b>0.054</b>	0.984	-0.004	0.146	-0.006	0.103	0.002
XON	<b>0.003</b>	<b>-0.013</b>	<b>0.000</b>	<b>-0.251</b>	0.104	-0.006	0.000	-0.158	0.366	-0.081	<b>0.000</b>	0.009

Bold format denotes significance at the 5 percent level.

level of confidence. Past volume and standardized durations seem to have the greatest explanatory power. Short durations and thus high trading intensity follow large returns and large trades (see columns for return and volume). Not surprisingly, the results for the trade dummy variable stress the relevance of correlation among trades. That is, trades not at the mid-quote are rapidly followed by other trades. The evidence for spread is weak. Spread is, in fact, significant for only a few stocks and the direction of its effect is not clear.<sup>19</sup>

Our results show preliminary evidence that incorporating feedback effects of returns, trades, and volume on time durations may improve the in-sample performance of the model. Yet, formulating and estimating a complete system, where prices and trades fully interact with spread, volume, and durations, is beyond the scope of this paper and a task for future research. Such research could ultimately provide more accurate impulse response functions. We therefore use the ACD—a time series model for time—and limit ourselves to a detailed empirical analysis of omitted variables in the ACD specification. Most importantly, these findings do not invalidate the results on the relevance of time durations for models of prices and trades. We hope our work will motivate future research to fill the open gaps.

#### IV. Conclusions

In this paper, we generalize the model suggested by Hasbrouck (1991) for the dynamics of trades and quote revisions. We suggest adding the time between consecutive transactions among the determinants of both the price impact and the autocorrelation of trades. We obtain the following main result. Short time durations, and hence high trading activity, are related to both larger quote revisions and stronger positive autocorrelations of trades. For instance, when a buy order is executed right after a previous order, it becomes more likely that it will be followed by another buy. We distinguish between a periodic and a stochastic component of time duration, and we show that the latter is mainly responsible for the above effects. Combining these with Hasbrouck's original results, high trading activity is coupled with large spreads, high volume, and high price impact of trades. This leads to the following scenario. When trading size is large and/or trading frequency is high, the liquidity providers revise upward their beliefs that an information event has occurred. Therefore, some of the liquidity traders may decide to postpone their trading and/or demand larger spreads, and the specialist adjusts prices more rapidly in response to trades. Active markets are thereby illiquid in the sense that trades have greater impact on price and higher informational content.

<sup>19</sup> The result of the test for the omission of spread alone suggests that spread tends to be positively correlated with durations.

In addition, the incorporation of time in modeling price dynamics has a range of empirical applications. For instance, it lets us directly measure how the price impact of a trade varies throughout the trading day and enables us to compute the time it takes for an average trade to induce a price revision of a given amount.

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