**ECON 120A , SPRING 2003 --- Prof. Ramu Ramanathan**

**ANSWERS TO PROBLEM SET#2**

I. This is a binomial case. n=25, and p=0.4. Let X=the number of patients who recover. The drug will be discredited if less than 10 patients recovered. The desired probability is

P(X≤9)= 1-P(X≥10)=1- $\sum_{10}^{25}(0.4)^x(0.6)^{25-x}$ =1-0.5754=0.4246.

II. Let $\overline{X}$ be the average income. Then by property 2.10a in Handout#2,

$\overline{X}$ ~N(μ, $\frac{\sigma^2}{n}$)=N(50,100/25)=N(50, 4).

We need P(47< $\overline{X}$ <52). If we subtract the mean and divide by the standard deviation, the resulting random variable has the standard normal distribution Z.

Therefore, the probability is equal to

P( $\frac{47-50}{4} \le Z \le \frac{52-50}{4}$ ) = P(-0.75≤ Z ≤ 0.5).

Using the symmetry of the distribution, we have

P(-0.75≤ Z ≤ 0.5)= P(0≤ Z ≤ 0.75)+ P(0≤ Z ≤ 0.5)=0.2734+0.1915=0.4649.

III.

1. $\int_0^\theta f(x)dx = \int_0^\theta \frac{1}{\theta}dx = \frac{1}{\theta}\theta - 0 = 1$

2. $E(X^m) = \int_0^\theta \frac{X^m}{\theta} dx = \frac{\theta^m}{m+1}$

3. E(X)= $\theta/2$, $E(X^2) = \theta^2/3$, $E(X^3) = \theta^3/4$, $E(X^4) = \theta^4/5$.

4. Var(X) = $E(X^2) - (E(X))^2 = \theta^2/12$,

Var(Y) = $E(Y^2) - (E(Y))^2 = E(X^4) - (E(X^2))^2 = 4\theta^4/45$,

Cov(X,Y)=E(XY)-E(X)E(Y)= $E(X^3) - E(X)E(X^2) = \theta^3/12$

$$\rho_{XY}^2 = \frac{(Cov(X,Y))^2}{Var(X)Var(Y)} = 0.9375.$$

5. $\rho_{XY}^2$ is still equal to 0.9375 since it doesn't depend on $\theta$.

IVV. 1.

| X | Prob. | X^2 | X^3 | X^4 | E(X) | E(X^2) | E(X^3) | E(X^4) |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.04 | 1 | 1 | 1 | 0.04 | 0.04 | 0.04 | 0.04 |
| 2 | 0.04 | 4 | 8 | 16 | 0.08 | 0.16 | 0.32 | 0.64 |
| 3 | 0.04 | 9 | 27 | 81 | 0.12 | 0.36 | 1.08 | 3.24 |
| 4 | 0.04 | 16 | 64 | 256 | 0.16 | 0.64 | 2.56 | 10.24 |
| 5 | 0.04 | 25 | 125 | 625 | 0.2 | 1 | 5 | 25 |
| 6 | 0.04 | 36 | 216 | 1296 | 0.24 | 1.44 | 8.64 | 51.84 |
| 7 | 0.04 | 49 | 343 | 2401 | 0.28 | 1.96 | 13.72 | 96.04 |
| 8 | 0.04 | 64 | 512 | 4096 | 0.32 | 2.56 | 20.48 | 163.84 |
| 9 | 0.04 | 81 | 729 | 6561 | 0.36 | 3.24 | 29.16 | 262.44 |
| 10 | 0.04 | 100 | 1000 | 10000 | 0.4 | 4 | 40 | 400 |
| 11 | 0.04 | 121 | 1331 | 14641 | 0.44 | 4.84 | 53.24 | 585.64 |
| 12 | 0.04 | 144 | 1728 | 20736 | 0.48 | 5.76 | 69.12 | 829.44 |
| 13 | 0.04 | 169 | 2197 | 28561 | 0.52 | 6.76 | 87.88 | 1142.44 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 14 | 0.04 | 196 | 2744 | 38416 | 0.56 | 7.84 | 109.76 | 1536.64 |
| 15 | 0.04 | 225 | 3375 | 50625 | 0.6 | 9 | 135 | 2025 |
| 16 | 0.04 | 256 | 4096 | 65536 | 0.64 | 10.24 | 163.84 | 2621.44 |
| 17 | 0.04 | 289 | 4913 | 83521 | 0.68 | 11.56 | 196.52 | 3340.84 |
| 18 | 0.04 | 324 | 5832 | 104976 | 0.72 | 12.96 | 233.28 | 4199.04 |
| 19 | 0.04 | 361 | 6859 | 130321 | 0.76 | 14.44 | 274.36 | 5212.84 |
| 20 | 0.04 | 400 | 8000 | 160000 | 0.8 | 16 | 320 | 6400 |
| 21 | 0.04 | 441 | 9261 | 194481 | 0.84 | 17.64 | 370.44 | 7779.24 |
| 22 | 0.04 | 484 | 10648 | 234256 | 0.88 | 19.36 | 425.92 | 9370.24 |
| 23 | 0.04 | 529 | 12167 | 279841 | 0.92 | 21.16 | 486.68 | 11193.64 |
| 24 | 0.04 | 576 | 13824 | 331776 | 0.96 | 23.04 | 552.96 | 13271.04 |
| 25 | 0.04 | 625 | 15625 | 390625 | 1 | 25 | 625 | 15625 |
| | | | | | 13 | 221 | 4225 | 86145.8 |

2. E(Y)= $E(X^2)$=221,

Var(Y)= $E(Y^2)-(E(Y))^2 = E(X^4)-(E(X^2))^2$ =37304.8

Cov(X,Y)=E(XY)-E(X)E(Y)= $E(X^3)-E(X)E(X^2)$ =4225-13*221=1352

3. $\rho^2_{XY} = \frac{(Cov(X,Y))^2}{Var(X)Var(Y)} = \frac{1352^2}{37304.8 * 52}$ =0.943