**Second-Year Advanced Microeconomics: Behavioural Economics**
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**Practice Problems on Behavioural Decision Theory: Present-Bias and Time-Inconsistency in Intertemporal Choice**

1. Consider a consumer with contemporaneous utility of consumption $u(c) = \frac{c^{1-\rho}-1}{1-\rho}$ for some constant parameter $\rho \in (0,1)$ in each of three periods. S/he discounts utilities that are one period in the future by the factor $\delta_1$ and utilities that are two periods in the future by $\delta_2$. (Note that these definitions imply that $\delta_1$ and $\delta_2$ shift with the passage of time. S/he has wealth W > 0, which s/he will consume completely over the three periods. S/he has access to a perfect capital market that allows her/him to borrow or lend at a constant rate $r > 0$. Thus her/his planning problem is:

$$max_{\{c_0,c_1,c_2\}}\ u(c_0) + \delta_1 u(c_1) + \delta_2 u(c_2)$$

subject to the constraint

$$W_0 = c_0 + \frac{c_1}{1+r} + \frac{c_2}{(1+r)^2}$$

(i) Write the first-order conditions that determine the optimal $c_0$, $c_1$, and $c_2$, and explain why they are sufficient for an optimum. (You need not solve them explicitly for $c_0$, $c_1$, and $c_2$.)

(ii) Give a definition of time-consistent planning (formal or informal, as you prefer; but precise). What condition on $\delta_1$ and $\delta_2$ insures that the consumer's planning is time-consistent? Explain.

2. (from the 2010 First-year M.Phil. final exam; note that it's "O'Donoghue")

In their paper, "*Doing it now or later,*" O'Donaghue and Rabin (1999) remark, "...economists should be cautious when exploring present-biased preferences solely with the assumption of sophistication."

(i) Exposit, briefly, the so called '$\beta - \delta$ model' used to analyze *present-biased* intertemporal preferences, including the idea of three (behavioral) types of agents usually invoked in the related literature, *time consistent (TC)*, *naifs* and *sophisticates.*

(ii) Explain the argument behind O'Donaghue and Rabin's remark.

(iii) With respect to the remark, some may say that the problem is not with the assumption of sophistication per se – people clearly have some degree of sophistication, what is problematic is that the model assumes a very rigid, extreme form of sophistication. Do you agree? How might one go about modeling a more nuanced form of sophistication?

3. Consider a consumer faced with a "vice" good like potato chips, which s/he is tempted to consume rapidly, with adverse future health consequences. The consumer can buy either a large (2-serving) or small (1-serving) pack at period 0. In period 1, s/he must then decide how much to consume. If she bought only the small pack, s/he consumes one serving. If s/he bought the large pack, s/he can consume two servings right away, or consume one serving right away and save another serving (which will then be automatically consumed in period 2).

Assume there is positive utility in period 1 from consumption, and negative utility in period 2 (a reduced-form for adverse future health consequences). Because the large size has some production economies it is cheaper, hence it yields higher immediate consumption utility. The Table below shows numerical utilities. (E.g. if s/he chooses to eat 1 serving from the large pack in period 1, s/he gets utility of +3 in period 2, and –2 in period 3 from the second pack.)

| Purchase decision<br>Consumption decision | Instantaneous utility in period 1 | Instantaneous utility in period 2 |
|---|---|---|
| Small<br>1 serving | 2.5 | -2.0 |
| Large<br>1 serving<br>2 servings | <br>3.0<br>6.0 | <br>-2.0<br>-7.0 |

Now consider a $\beta$-$\delta$ quasi-hyperbolic framework. For simplicity assume $\delta = 1$. Analyze the optimal consumption decisions of three types of agents: Exponential ($\beta = 1$, $\hat{\beta} = 1$; naïve hyperbolic ($\beta < 1$, $\hat{\beta} = 1$); and sophisticated hyperbolic ($\beta < 1$, $\hat{\beta} = \beta$).

For each type of agent, figure out, as a function of $\beta$ and/or $\hat{\beta}$ if they matter:

(i) What will s/he expect at period 0 to consume in periods 1 and 2, contingent on buying the large or respectively the small package? (Recall that naivete means that in period 0 the agent expects to be time-consistent in periods 1 and 2.)

(i) Given your answer in (i), which package will each type of agent purchase in period 0?

(ii) Given the optimal package in (ii), how much will each type of agent consume in period 1?

(iii) Which (if any) of the types' plans in (i) are actually violated in (iv)?

(iv) Suppose, at a price of P > 0, agents can purchase "commitment" (e.g. pre-packaged portions), limiting them to consume only 1 of the 2 servings in the large pack in period 1. Which type(s) of agent would, in period 0, pay P > 0 for such commitment, and what is the most each such type would be willing to pay?

4. As question 3 indicates, an important empirical demarcation between naïve and sophisticated hyperbolic agents is whether they will pay in advance for planned self-control (à la Ulysses and the Sirens). Give an example of external self-control that is *voluntarily* chosen by agents (other than those discussed in the slides or lectures).

5. a. Consider a quasi-hyperbolic naïf with $\delta$= 1 and β = ½ but $\hat{\beta} = 1$. Time is indexed t ϵ {0,1,2,…}. The naïf must finish a project by a deadline T < ∞ at the latest. In period t the project costs $(\frac{3}{2})^t$ utils to finish. (There is no time discounting; just the increasing cost of doing it later.) Commitment is impossible. When will the naïf do the project?

b. Now consider a quasi-hyperbolic sophisticated agent with $\delta$= 1 and $\beta = \hat{\beta} = \frac{1}{2}$, with everything else as in part (a). Recall that in this dynamic setting, an agent's behavior must be characterized by a complete contingent plan (or strategy). Prove the following two claims:

(i) If T is an even number, then a sophisticate will do the project in any even period (that is, if s/he has not already done it) but not in any odd period.

(ii) If T is an odd number, then a sophisticate will do the project in any odd period (if s/he has not already done it) but not in any even period. (Note that (ii) is a corollary of (i).)

c. Now consider a quasi-hyperbolic *partially*-naïve agent with $\delta$= 1, β = ½, but $\hat{\beta}\epsilon(\beta, 1)$. That is, the agent incorrectly believes that her/his future selves will have beta parameter $\hat{\beta} > \beta$, when in fact it will be $\beta$. Assume T is even, with everything else as before.

(i) Solve for the lowest value of $\hat{\beta}$ for which there is an equilibrium in which the agent does not do the project until period T. (Hint: Consider the case $\hat{\beta} = \frac{2}{3}$ and show that this is the key threshold value.)

(ii) Show that when $\hat{\beta} < \frac{2}{3}$, the project is completed in period 0.