

What's Luck Got to Do with It?  
A Simple Hyperbolic Discounting Utility Model of Gambling Behavior

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## ABSTRACT

Existing theories in utility analysis are shy of providing adequate explanations for certain economic behaviors such as gambling. A simple model is proposed to account for the potential effect that certain mental biases can hold over decisions made during play. When cognitive factors are assessed into the utility framework, a better understanding of why gamblers may choose to continue betting even though expected payoffs and probability of winning are less than ideal can be achieved.

*“As for myself, I lost every penny I had, and very quickly, too. I put two hundred gulden on even right off the bat, and won, bet another five and own again, and so two or three more times. I think I must have picked up about four thousand gulden in five minutes or so. This is where I should have quit, but some kind of strange sensation built up in me, a kind of challenge to fate, a kind of desire to give it a flick on the nose, or stick out my tongue at it. I placed the largest bet allowed...”*

—Dostoyevsky, *The Gambler*

## I. INTRODUCTION

Everyone, whether aware of it or not, takes chances every day of his life. “If I cross the street, chances are I will make it across safely”, “Past experiences tell me there is a good chance I will wake up after sleeping” and the like are essentially gambles we place on common day by day events that their odds of happening are probable, and these outcomes thereby constitute beliefs base upon which decisions are formed. Chance therefore governs our world, and yet, is incidentally governed by laws of its own. Early studies of chance were primarily a mathematics concern but as this science expanded, the mathematician soon became statistician and from there evolved into econometrician (Levinson 18.) The rapid development of this discipline proves just how ubiquitous a role probability plays in science and has managed to permeate a wide variety of subjects over the course of just this century. Mathematics can claim credit to have pioneered in treating chance as a scientific quandary but even before the mathematician was toying with probability in his diapers, there was the gambler.

Though not received by society with as much fanfare as the former, the fact remains that it was the gambler who first put probability to good use by acknowledging its existence, and

through practicing his craft, explaining away its wild tendencies with his own rationalization.

Perhaps luck must be a lady because with every roll of the dice and every turn of the wheel, it always seems to change mood or heads in an uncontrollable direction that is almost impossible to predict. Despite or possibly due to its high variance, the typical gambler would posit chance as his ticket to wealth and can invariably be harnessed to his benefit if understood properly.

Without a doubt, probability is master, luck its lord, and the gambler their servant for it is often the case in which gamblers chain themselves into believing they understand how chance works that paves the road to ruins, not riches.

Of course, gambling is not just a personal or trivial pursuit. For centuries across cultures gambling is integral to the functions of everyday life, both directly and indirectly. In Japan and China social decisions were determined by the play of chess games such as Go and Wei Kei (Dickerson 8.) In the case of some German tribal leaders at the forefront of a Roman invasion, however, gambling brought politics to its demise by luring players into betting on “their wives, their children and ultimately themselves into captivity” (Dickerson 8.) Indeed, this blinding effect over one’s senses that gambling can cast is certainly mystifying, and to the economist it is almost unnatural.

If people are rational, and economists would like to believe so, then a continual losing streak on the slot machine or by the roulette wheel should give ample reason to quit. How could addiction to any form of gambling be possible if the prospect of winning is slim and its expected value negative? The attraction of gambling thus lies in its get-rich-quick appeal but whenever easy wealth can be created with luck, so too could poverty. This reality is even more apparent at the casinos where millions can be won and lost in matter of minutes. Obviously, the casino business is not lucrative by chance because the games are designed so that odds of winning are

skewed in favor of the house. Therefore, if gambling is generally a losing proposition and the house almost always win, why play?

The success of Bally, the Bellagio, MGM Grand and many others in Las Vegas then is evidence against the theory of rational behavior. According to Kahneman and Tversky, the process involved in decision-making for the layman takes into account gains and losses rather than welfare (Kahneman and Tversky 274). Since economic activities are driven by monetary incentives, gambling participation is consistent with economics in that acquiring some financial reward in every play is possible. In addition, the prospect theory proposed by Kahneman and Tversky assumes loss aversion to be another component of value accounting. However, this tendency seems to not hold when applied to gambling. If losing \$10 is more unfavorable than gaining \$10 gives utility, then loss through gambling should induce cessation of play. On the contrary, typical gambling behavior shows that the frequency of playing is associated with higher losses and vice versa (Dickerson 134.) Obsessive or neurotic gambling presents an even greater problem to the assumption of rationality in economic studies where expected value seems to be disregarded completely.

Despite its seemingly counterintuitive attribute in light of economics, gambling has historically been popular, and given the steady rise of the casino industry, perhaps more so now than ever before. The puzzling implications behind gambling behavior therefore challenge the predictive power of standard economic theories by showing that vintage utility models are not always applicable. Such deviation from conceptual norms then qualifies as reason for continued development of our discipline, and further refinement on the literature is necessary if economic activities like gambling cannot be adequately explained. As the excerpt from *The Gambler* reveals, better understanding of how a gambler regard risk and payoff lies in his thought process

at the moment before betting takes place. From a strictly normative assessment of his gains, the gambler from the story admits that it would have been in his best interest to quit after winning the large sums. More important, taking the next bet was not intended by the gambler but based on a whim. If decisions are vulnerable to alter under sudden impulse, then the thought process of a gambler at every round should be examined separately rather than in aggregate. In trying to study gambling however, is the idea that economic choices are influenced by cognitive variations a hasty presumption?

Psychological studies on gambling behavior discover that crucial to the persistence of gambling participation is the perception of win held by the player. "When individuals cannot directly modify their environment, they modify their perceptions in order to feel in control of the outcome" (Coulombe et al. 243.) Other experiments in Psychology find that gamblers tend to believe in the maturity of chances by believing that a streak of undesirable outcomes will soon produce better luck as time progresses. Also known as *gambler's fallacy* or more technically as *negative recency*, this line of reasoning motivates the individual gambler to continue betting even though he or she may be chin-deep in debt (Altmann and Burns 1.) Gamblers hold dear to their erroneous perception of chance as this attempt to control its randomness induces them to stay in the game even when expectation of win is low (Sylvain, Ladouceur and Boisvert 727.) Under this cognition it appears that information from past events can be misattributed by gamblers while making decisions. In this sense, the gambler's perception of performance and payoff may differ from reality because mental bias distorts his interpretation of future outcomes. Additional psychological mechanisms like denial and rationalization of costs are also often used in order to continue play (Abt et al. 7.) Such cognitive style is characteristic of gamblers rather than non-

gamblers and the role biased beliefs play as a mental rule can potentially explain for the persistence of gambling.

Since perfect information is required to make the optimal decision, this presence of mental bias belies facts regarding actual odds of winning, stakes lost and won, expected payoff, and skills involved—all of which are factors that the gambler weighs during play. Imperfect information poses problem to decision theories because it is inconsistent with the assumption of perfect information traditional in economics. Situations that involve imperfect information complicate game theoretic principles in that decisions can potentially deviate from equilibrium when contradicting information held by the individual provoke actions that may not have been planned otherwise (Aumann, Hart and Perry 97). As a result, strategies taken by the individual might not be optimal in instances where imperfect information prevails such as gambling.

The case of the absentminded driver is one famous application of this alternative supposition. This story tells of an individual who, after gulping down one two many drinks at the bar, must plan out his route home when closing time nears. In order to get home he must exit at the second intersection of the highway and not the first because doing so will land him in a “disastrous area” (Piccione and Rubinstein 7.) Going pass the second exit will never lead him home for he cannot go back, but can then decide to stay at a hotel at the end of the road. The problem presents itself when, upon reaching an intersection, the individual cannot recognize which one it is and whether or not he has passed the first. Due to his absentmindedness, he cannot perfectly recall the necessary information that would get him home and may choose an action that he otherwise would not have opted for. If imperfect recall influences the individual’s decisions both at the planning and action stage, then the paradox here lies in the conclusion that: even when there is no new information, the optimal strategy for the decision-maker will change

when he reaches a subsequent stage. Optimal strategies for driver and gambler with imperfect information must necessarily be different from those in the instance of perfect information.

As Piccione and Rubinstein pointed out, commitment to the optimal decision made in the planning stage is not possible when the driver suddenly finds himself lost upon reaching an intersection (Piccione and Rubinstein 8.) In addition, the driver was not exposed to new information as he leaves the planning stage to arrive at the action stage—the intersection. Therefore, if his preferences did not change, the optimal strategy that he originally decided on must still apply. However, sticking to plan A may not be optimal when the driver reaches the next stage where action must be carried out. Based on his current belief—that he might or might not be at the right intersection—the driver seeks to maximize his expected payoff, which could require him to deviate from the optimal strategy in the planning stage. This conflict between choosing the optimal strategy and maximizing expected payoff given certain beliefs is central to the problem that is born by imperfect recall (Piccione and Rubinstein 8.)

A different treatment of mental bias can also be found in behavioral analysis. Two most relevant theories on which this paper relies have been proposed to describe how temporal discounting of future outcomes may affect choices. The prospect and hyperbolic discounting models incorporate new ideas into the classic utility paradigm by factoring in time-discounting as a major reason behind decision inconsistency. Such premise holds valuable implications for studying gambling behavior because it allows decisions to change as the individual acts on his choices. If “addictive” behavior is the consequence of a series consisting of short-run actions that are actually chosen on impulse and not necessarily planned, then gambling addiction can only be observed. By drawing from the foundation on time-discounting presented in these two models, this paper hopes to show that a gambler’s choice to take the next bet is driven by time-

inconsistent preferences, and how the presence of mental bias can induce irrational or addictive gambling even when loss might greatly exceed gain.

## II. LITERATURE REVIEW

Before delving into further analysis of this issue, however, a brief review of the literature is necessary. Since gambling decision should be examined as a time-inconsistent behavior, two main theories that deal with such assumption can be found in the *hyperbolic discounting utility* and *prospect theory*. The former had been adapted and made salient by David Laibson in treating time-discounting rates as a changing phenomenon over time. Unlike its predecessor, the exponential discounting theory, this new and improved model proposes that people tend to discount the future at a changing rather than constant rate. “Hyperbolic discount functions are characterized by a relatively high discount rate over short horizons and a relatively low discount rate over long horizons” (Laibson, 445.) At time  $t$  a person’s preference for  $t+1$  may not hold when  $t+1$  arrives. Preferences are then time-inconsistent since greater weight is given to the immediate future over the far future (Rubinstein 1208.) Since discount rate is not exponential and is relatively high in the short-run but relatively low in the long-run, a discrete-time quasi-hyperbolic discount function can be represented by attaching a  $\beta\delta^t$  rate as time progresses. There is evidence from laboratory experiments supporting this theory that discount rates are rather hyperbolic in nature. Studies conducted in Biological Psychology find that not only humans, but animals, also exhibit such type of discounting (Green and Myerson 496).

The hyperbolic model fits the prospect theory by suggesting that utility function should be hyperbolic. Under this theory, gains and losses are calculated according to a specific reference point and not in aggregate. While the gain function is concave and the loss function is convex, both exhibit diminishing sensitivity. In addition, the loss-aversion clause indicates that loss hurts

more than gain benefits (Tversky and Kahneman.) Since both models lay out the basic assumptions of time-inconsistency, the implications behind their ideas are useful to gambling behavior analysis by providing a solid framework of present-bias preferences as a result of hyperbolic time discounting. This paper would like to tailor these two theories to the context of gambling analysis by showing how the existence of cognitive bias can further strengthen a gambler's tendency to prefer present over future outcomes. In doing so, better understanding of how decisions are formed and how gambling may still very much be an economic activity.

### III. THE MODEL

The existence of cognitive bias influences our current knowledge and beliefs on how we remember and interpret past events. Psychological studies find that gamblers do possess certain mental biases that tend to distort their perception of the game and can lead them to continue playing even after amassing huge losses. By assuming hyperbolic discounting utility, this model allows the gambler to be time-inconsistent. That is, he may not want to bet in future round  $n+1$  when he is in current round  $n$ , however when he reaches round  $n+1$ , he places the bet because his preference for immediate gratification will have set in. This current round then becomes the new reference point by which he weighs his gains and losses as he forms his decisions for the future. Therefore, cognitive bias plays devil's advocate by falsifying the gambler's reality and induces him to always want to place the bet as he reaches the next round. Addictive gambling is the observed behavior if the gambler continues to have time-inconsistent preference at every round.

In order to model gambling behavior, addictive or continuous play is assumed to be similar to the case in which the gambler always chooses to delay quitting when the next round arrives. Since biases distort information, hyperbolic discounting makes gambling in the next

round more attractive. Time-inconsistent preference can then result in the “procrastination” of quitting, which can help explain why people are observed to be “addicted” to gamble even when cost is high. Whether or not a person will delay quitting and how long such delay lasts are determined by his bias.

Our model adopts the hyperbolic discounting model (Laibson and O’donoghue & Rabin) as base for examining gambling decisions with mental bias.

$$U(u_n, u_{n+1}, \dots, u_N) \equiv E[U(c) + \beta \sum \delta u(c)]$$

where  $\delta = [0, 1]$

Time-consistent:  $\beta = 1$

Time-inconsistent:  $\beta < 1$

*Definition 1:*

Cost function:  $C = (c_1, c_2, c_3)$

How much the gambler must or is willing to bet.

Reward function:  $W = (w_1, w_2, w_3)$

How much he receives from the gamble

$$U = (W - C)$$

A gambler’s utility function is defined to be the net difference of wealth that he acquires from gambling. His utility, however, is attached to two distinct discount factors that represent time discounting and hyperbolic discounting. If both  $\beta$  and  $\delta$  are less than unity, then the gambler has time-inconsistent preferences where the next round is mostly likely to carry more weight.

*Definition 2:*

where  $b_n^{\wedge}$ : the gambler’s belief subjected to his own perception of the game at round  $n$ .

$I_n^{\wedge}$ : information set subjected to his own perception of the game at round  $n$ .

$\rho^{\wedge}(n)$ : probability (of the game or bet) as perceived by the gambler at round  $n$ .

$r^{\wedge}(\cdot)$ : perceived outcome.

Types of gamblers:

**A:**  $b^n[I_n]$

This type is considered to have perfect perception. His belief about the game regarding risk, chance, performance and payoff is based on full information (that is not distorted.)

**B:**  $b^n[I_n \mid \beta r^{n+1}]$

This type hyperbolically discounts betting outcomes. His evaluation of the immediate next round is influenced by how much he discounts the future and can therefore be time-inconsistent with his preferences when making decisions.

**C:**  $b^n[I_n \mid \rho^n(n)]$

This type is possessed by a bias known as *gambler's fallacy*. Even though the probability of gambling outcomes is often independent, he nonetheless believes that chance would eventually mature with time.  $\rho^n(n)$  represents probability as perceived by the gambler to be a (linear) function of the number of gambling rounds played.

**D:**  $b^n[I_n \mid r^{\wedge}(T^{\wedge})]$

This type suffers from the illusion of control. He believes that his skill for the game increases over time and betting outcomes thus improve accordingly.  $T^{\wedge}$  represents his perceived talent and expected result of the game is a positive (exponential) function of talent.

The gambler forms decisions base on his belief in the game. This belief depends on what kind of information is available to him. Where mental bias exists, the gambler's information set  $I_n$  is represented to be conditional on another term. **A** is the only type that has full access to perfect information and whose belief can be considered time-consistent. **B** is not formally biased but since this type hyperbolically discounts the future, this tendency can be represented as a bias

in its own right. **C** and **D** both possess true cognitive biases that are found common in gambling behavior. The gambler's fallacy and illusion of control are perhaps the most typical form of cognitive rationalization. While the former reasons chance to be influenced over time (number of rounds played over time), the latter attributes not only time but personal skills to the effect.

"Illusion of control implies a perception of situations involving chance as resembling situations involving skills and that the more a situation of chance resembles a situation of skill, the more it will induce a skill-oriented behavior in the individual" (Ladouceur et al. *Illusion* 48.) In a situation of chance such as gambling, events are generally independent of one another. However, gambling rationale tends to link probability to some behavior as though chance always dictates any series of gambling rounds by some predictable pattern. Studies in Psychology reveal that holding erroneous beliefs is a predominant characteristic among gamblers and when they get to actively make decisions, expected win is higher (Dickerson 30.)

These misconceptions are believed to be determining factors in the development and maintenance of gambling habits (Ladouceur 557.) In addition to these two biases, another cognitive assumption made for type **D** rests on the idea that people are in general confident and that they hold positive self-image of themselves (Santos-Pinto and Sobel 1.) Thus,  $r^{\wedge}(T^{\wedge})$  is best presented as an exponential function because people tend to hold own ability in relatively higher regard. Thus, any personal characteristic that can be attributed to influencing performance will cause it to rise much faster than other impersonal factors such as number of rounds played. While it is entirely possible for a gambler to have two biases, this paper will only focus on attributing one bias per type. The intuition behind incorporating multibiases is not yet clear to this study because possessing two biases, as opposed to one, might or might not decrease the degree of influence such biases impose on reasoning. On one hand, the gambler with two biases

might prove to be less time-inconsistent due decreased weight each bias now has over information. Or, the level of inconsistency can lead to equal or greater time-inconsistency.

*Definition 3:*

where B: Bet  
 NB: Not bet

Strategy function:  $S = (s_1, s_2, s_3)$   
 (B, NB)  $s_n$

There are two basic strategies available to the gambler and can decide to gamble(quit), in which case action B(NB) would be chosen. He can start out intending to play so many rounds or quit after he has won or lost a certain amount of winning. When the gambler exhibits memory bias, however, his optimal strategy in the planning stage might not be so relevant now that he discovers he must make a choice and the information available to him does not correspond to reality. That is, if the gambler lost more than he had won in past rounds and fools himself into believing that a series of bad luck will mature into good, then this judgment bias denies him complete information necessary to make the rational or optimal decision. If the gambler had full information, meaning there is no cognitive bias, his optimal strategy might be to quit since he is in the red. The presence of imperfect information leads him to reason differently when he cannot correctly interpret earlier events, and at the action stage quitting might not be so appealing. Once again, mental bias creeps into the decision-making process by preventing the individual from choosing the optimal strategy.

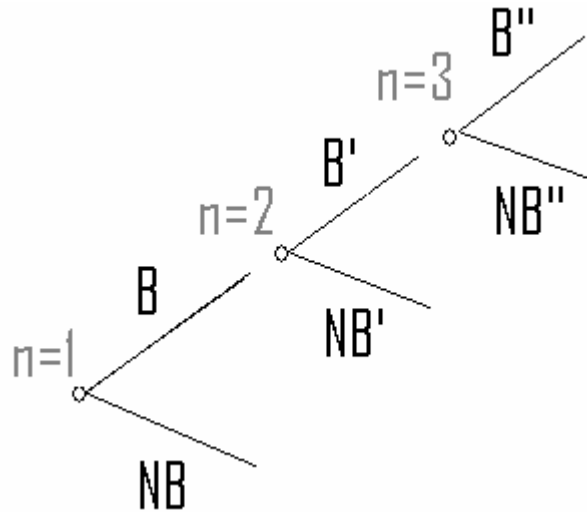


Figure 1. Game Tree Representation of Gambling Decisions

The extensive form of the decision problem facing the gambler is illustrated with  $n(=3)$  action stage(s) and the gambler must decide between two actions: to bet and not to bet. The initial stage at which the game begins is identified as 1 and each information node henceforth stands for a new round that the gambler must make a choice. The gambler's decision to bet or not to bet will depend upon his evaluation of the payoff that he expects to receive with continuing against that of quitting.

*Definition 4:*

$$\mathbf{A:} \begin{matrix} s_n = B & \text{if } w_n > c_n \\ \text{NB} & \text{if } w_n < c_n \end{matrix}$$

$$\mathbf{B, C, D:} \begin{matrix} s_n = B & \text{if } w^{\wedge}_n > c_n \mid b^{\wedge}_n \\ \text{NB} & \text{if } w^{\wedge}_n < c_n \mid b^{\wedge}_n \end{matrix}$$

When deciding on the best strategy for the next round, the gambler takes into account his payoff, which is conditional on expected beliefs about the game in the current round. Not only does the expected payoff of the optimal strategy need to be larger than the alternative, it

must also costs. Due to mental bias, the gambler can only form decisions based on how he perceives the state of the game to be, not how it actually is. His information set is distorted by the bias and such imperfection can lead the gambler to persist with gambling even though, in actuality, he may be in the red. Reward and cost therefore, are subjected to personal belief for type **B**, **C** and **D**.

*Definition 5:*

- (i) Bias distorts information that the gambler relies on to decide whether to bet or not in the next round. If information is not perfect, then his perception of the game regarding risk, chance, performance and payoff can lead him to play a suboptimal strategy.
- (ii) If no bias exists, the chosen strategy is always optimal.

*Proposition 1:*

- (i) If  $\rho^{(n)} \approx r^{(n)}$  and  $\rho^{(n)} > 0$ , then  $B \geq NB$ .
- (ii) If  $r^{(T^*)} > 0$ , then  $B \geq NB$ .

We assume the perceived outcome held by type **C** to be similar to its probability function where playing more rounds will increase the probability of winning and thus, the reward itself. If the gambler expects positive probability and reward, then he will choose strategy **B**. Likewise, the bias for type **D** will lead him to choose strategy **B** when his perceived outcome is greater than 0. Since perceived outcome is a function of talent, which is also a positive function of number of rounds played, then playing in the next round is the dominant strategy for this gambler.

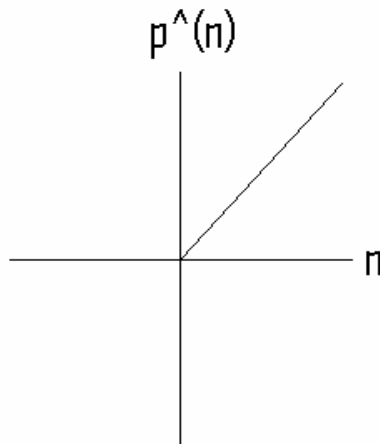


Figure 2. Perceived probability as a function of rounds played.

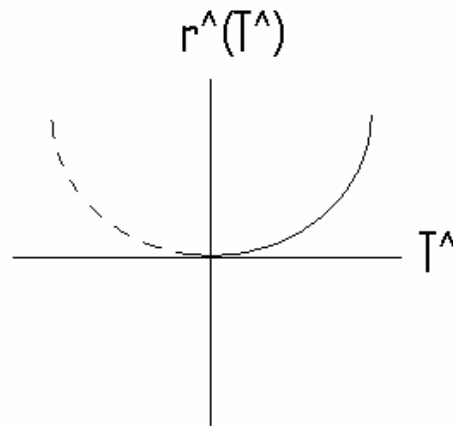


Figure 3. Perceived outcome as function of talent.

In both figures, the number of rounds played  $n$  greatly affects the dependent variables  $\rho^{(n)}$  and  $r^{(T^A)}$ . Although  $n$  does not directly influences  $r^{(T^A)}$ , it does so through  $T^A$  when  $T^A$  is understood to be positively related to  $n$ . This assumption dwells on the fact that gamblers believe talent to improve over time and this can only happen when more rounds are played. The presence of these mental biases make it attractive for the gambler to choose strategy B when perceived probability and outcome are greater than 0.

*Definition 6:*

where  $n$ : round that  $s^*$  should be played  
 $\eta$ : round that  $s^*$  is actually played

$\beta$ : hyperbolic discount factor  
 $\delta$ : exponential discount factor

Immediate cost and reward of playing  $s^*$

$$U_\eta = \begin{cases} w - c & \text{if } \eta = n \\ \beta\delta(w - c) & \text{if } \eta \neq n \end{cases}$$

If the gambler actually plays the optimal strategy  $s^*$  in the appropriate round  $n$  his utility is simply the net difference of his gain and loss. On the other hand, if  $s^*$  is played in some other round that is not  $n$  but  $\eta$ , then the gambler is time-inconsistent with his preference by playing the optimal strategy intended for that current round at another time. For simplicity's sake, it is assumed that by executing  $s^*$  in round  $\eta$  the gambler is delaying that particular strategy and not pre-playing it.

*Definition 7:*

$$\begin{aligned} \text{Max } U \mid b^{\wedge}_n = \max (w - c) \mid I^{\wedge}_n \\ \text{Subject to } I^{\wedge}_n = I_n \\ \qquad \qquad \qquad = \beta r^{\wedge}_{n+1} \\ \qquad \qquad \qquad = \rho^{\wedge}(n) \\ \qquad \qquad \qquad = r^{\wedge}(T^{\wedge}) \end{aligned}$$

Gambling remains a maximization problem that is conditional on the gambler's information set. The gambler seeks to maximize utility by optimizing his net reward based on his respective information set. If cognitive bias exists, then these mental handicaps act as constraints in preventing him from accurately maximizing utility.

*Definition 8:*

$$U_\eta = \begin{cases} (w - c) & \text{if } s_\eta^*(NB) = s_n^*(NB) \\ \beta\delta(w - c) & \text{if } s_\eta(NB) = s_n(NB) \\ \beta\delta^{(\eta-n)}(w - c) & \text{if } s_\eta(NB) > s_n(NB) \\ \beta\delta^{(\infty-n)}(w - c) & \text{if } s_\eta(NB) = \end{cases}$$

A gambler delays quitting if he plays strategy  $s(B)$  when there exists strategy  $s(NB)$  that is  $s_n^*$  optimal and  $\eta > n$ .

Since  $b_n$  dictates  $s_\eta$ , then  $s_\eta \neq s_n^*$  for types **B**, **C** and **D**.

Drawing from the idea of procrastination as a strategy to delay action proposed by Ted O'Donoghue and Matthew Rabin, persistent or addictive gambling can be shown as the result of the gambler's decision to delay quitting or playing strategy NB. If the optimal strategy for the gambler is to play NB in round  $n$ , then he delays the action if NB is subsequently played in round  $\eta$  (O'Donoghue and Rabin 137.) This inconsistency in playing the best strategy is possible for types **B**, **C**, and **D** because their biases may lead them to play a strategy in round  $n$  that need not be optimal. If  $\eta = n$ , then type **A** will play the optimal strategy when he should in the right round. Only type **A**'s response can be considered strictly optimal while another biased type, say **B**, can still choose to play the best strategy on time but whose utility function will correspond to his bias. The discount factor will increase by the difference of time delayed ( $\eta - n$ ), which is how long it takes  $s(NB)$  to be played and where  $s(NB)$  is never played so that quitting is put off indefinitely, the degree of time-inconsistency denoted by the discount factor is raised to infinite power.

*Proposition 2:*

- (i) Mental bias for types **B**, **C**, and **D** induces them to have time-inconsistent preference. The gambler will delay quitting even though it might be the optimal strategy to play.
- (ii) Without bias, the gambler will quit when it is the best response.

*Example 1:*

$$w_1 > c_1 ; w_2 < c_2$$

$$s^*_2(NB)$$

$S^A(B, NB, NB)$	$\eta = n = 2$
$S^B(B, B \text{ or } NB, NB)$	$\eta = 2, 3$
$S^C(B, B, NB)$	$\eta = 3$
$S^D(B, B, B)$	$\eta = \infty$

Every type chooses to bet in round 1 when expected reward is higher than cost. However, if in round 2 the cost of gambling outweighs expected reward, type **A** will definitely choose to quit by playing action  $s_2(NB) = s^*$  while type **B** may or may not choose the optimal strategy. Depending on the value of the discount factors, type **B** could end up quitting in round  $n$  or delaying until  $n+1$ . The reason for this uncertainty in type **B**'s behavior lies with our assumption that hyperbolic discounting is a bias but not strong enough to induce definite delay like the other two typical gambling biases. Although hyperbolic discounting allows an individual to be time-inconsistent, it does not directly imply that his preference for the next round will always be skewed towards playing  $s(B)$ . Type **B**'s perception of the game is therefore not necessarily biased and is still perfectly capable of choosing the optimal strategy in the appropriate round. Type **C** will delay playing the optimal strategy  $s(NB)$  until period 3 but type **D** will put off playing  $s(NB)$  forever.

Since type **D** perceives the probability of the game to be increasing as a result of personal skills, this type would most likely believe in a higher probability of winning if he plays the current round, the next round and the round after that. If there is any type that would procrastinate and never opt to play  $s(NB)$ , it would be type **D**. His bias generates more distortion because personal belief is linked to characteristics that are endogenous and thus possible to control.

*Proposition 3:*

- (i) Hyperbolic discounting increases the attractiveness of playing  $s(B)$  in the immediate next round.

If the reward is adequately high, it is possible that hyperbolic discounting alone can make

the next round seem tempting enough for play. This proposition applies to type **B** most where his only bias draws from his discounting tendency. While the bias may not be enough to ensure definite preference for playing  $s(B)$ , it does increase his likelihood to do so. In addition, the discounting assumption only strengthens type **C**'s and **D**'s inclination to delay playing  $s(NB)$  either for awhile or forever.

*Example 2:*

$$W_{n=1,2,3} < C_{n=1,2,3}$$

$S^A(NB, NB, NB)$	$\eta = n = 1$
$S^B(B \text{ or } NB, B \text{ or } NB, NB)$	$\eta = 1, 2, 3$
$S^C(B, B, NB)$	$\eta = 3$
$S^D(B, B, B)$	$\eta = \infty$

If betting is not a winning proposition in any period, type **A** will not choose to gamble at all, type **B** may or may not choose to enter the game in period 1 and 2. The strategies will be the same for types **C** and **D** because their biases always require that  $n \geq 1$ .

*Proposition 4:*

- (i) Variation in the prize structure encourages continual play by enticing the biased gambler to aim for the larger reward.

A gambler is as ambitious as his bias allows him to be. When expected payoff varies in size, the unbiased gambler, namely type **A** and to some extent type **B**, will consider other aspects of the game such as probability and chance. If the probability is low and expected payoff is negative on average, then they will choose to quit. For type **C** and **D**, however, the choice between  $s(B)$  and  $s(NB)$  is unambiguous. Clearly, the respective cognitive bias for **C** and **D** naturally requires that betting takes place because outcome is dependent on the number of rounds played. In order to maximize expected payoff, quitting seems irrational when there exists the

possibility of gaining a positive reward. Even though reward may average in the negative, type **C** and **D** will opt to gamble with  $s(B)$  as long as there exists one large enough prize.

*Example 3:*

$$w_2 > c_2$$

$$w_2 = c_2$$

where  $w_2 = 2, -2$

$$c_2 = 1$$

The presence of cognitive bias will make betting attractive even when reward may be negative on average. Recall that each type of gambler seeks to maximize utility while being subjected to his respective bias:

$$\text{Max } U \mid \hat{b}_n = \max (w - c) \mid \hat{I}_n$$

$$\text{subject to } \hat{I}_n = (\cdot)$$

For type **A**,

$$\sum(w - c) = (2 - 1) + (-2 - 1) / 2 = -1$$

$$s^*_2 = s_2(\text{NB})$$

For type **B**,

$$\beta \sum \delta(w - c) = \beta \delta (-1) \geq -1$$

$$s^*_2 = s_2(\text{B}) \text{ or } s_2(\text{NB})$$

For type **C**,

$$U = \beta \delta [ \rho^{\wedge}(n) + (w - c) ]$$

$$\text{where } \rho^{\wedge}(n) = 2n$$

$$\partial U / \partial w = \beta \delta$$

$$w^{\wedge*} > 0 \text{ if } n \geq 1$$

$$\begin{aligned} \beta \sum \delta [r^{(n)} + (w - c)] &= \beta \delta [(2(n) + 1) + (2(n)-3) / 2] \\ &= \beta \delta [(2(2) + 1) + (2(2)-3) / 2] = 3 \end{aligned}$$

$$s^*_2 = s_2(B)$$

However,

If the gambler decides to not quit in round  $n=1$ ,

$$\beta \delta [(2(1) + 1) + (2(1)-3) / 2] = 1$$

then his payoff is significantly lower and if the prize structure varies enough, then he may be in the red if he decides to quit even when average expected payoff is negative.

For type **D**,

$$U = \beta \delta [r^{(T^*)} + (w - c)]$$

where  $T^*(n) = n$

$$r^{(T^*)} = n^2$$

$$\partial U / \partial w = \beta \delta$$

$$w^* > 0 \text{ if } n \geq 1$$

$$\begin{aligned} \beta \sum \delta [r^{(T^*)} + (w - c)] &= \beta \delta [(n)^2 + 1) + ((n)^2-3) / 2] \\ &= \beta \delta [(2)^2 + 1) + ((2)^2-3) / 2] = 4 \end{aligned}$$

$$s^*_2 = s_2(B)$$

Not betting, then, becomes irrational because it will definitely leave the gambler in the red and in order to lower the price of risk the game should continue. Similar to the case for type **C**, this type will continue gambling the higher  $n$  becomes and be more likely to take the next bet as the expected payoff rises exponentially, irregardless of the low average payoff.

In addition to expected payoff, chance plays an important role in any gambling game. If the gambler rationalizes chance by believing that poor luck will soon mature as long as he

persists playing, then this attempt to impose control over the probability of the game largely contributes to information bias. When the gambler perceives chance to be controllable, then the relationship between chance and number of rounds must be positive. The bias effect makes the gambler believe that chance will only increase or improve if he sticks to the game by playing more rounds. By choosing  $B$  in the next round the gambler increases the chance of raising his payoff while choosing  $NB$  will cut off any chance of improving his luck.

Imperfect recall requires that more rounds must be played if the gambler wants to maximize his future expected payoff and the strategy  $s(B)$  often dominates the alternative,  $s(NB)$ . Therefore, in order to maximize utility, the gambler must increase  $n$ . Given that the gambler's decision to participate in the game is based on subjective beliefs and not perfect reality, information held at each round need not and are not accurate due to the effect of cognitive bias.

#### IV. DISCUSSION AND CONCLUSION

It is not entirely obvious whether or not the delay of quitting is guaranteed under the effect of the illusion of control bias. The idea that quitting long past the appropriate round is closer to reality, since the different biases presented in this paper can only reveal *when* a gambler is more likely to quit rather than *whether* or not he does at all. The arguments presented in this paper are deficient in more ways than one. Lack of a refined mathematical representation of gambling behavior makes the implications proposed here less credible. But despite this problem, it remains asserted that gambling studies should incorporate mental bias into their analysis. By allowing for personal bias to be endogenous in the decision framework, more insights can be shed on how people in general and gamblers in particular, assess expected payoff in environments where probability and chance muddle economic certainty. Although the reality of

prior rounds signifies true odds of winning, rationalization of chance forces the player to ignore and forget these “signs”. Just like the absentminded driver who forgets the road signs that would lead him home, so too the gambler forgets the signs that would get him the largest winning. Therefore, imperfect recall can be applied because of such reasoning, which is crucial to explaining the decision-making behavior that would otherwise seems illogical.

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